

# THE CALCULATION OF STAGE EFFICIENCY AND ITS APPLICATION TO DIFFUSER DESIGN

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## Abstract

The actual mechanism controlling the extraction of sucrose from cane in diffusers is still in dispute. It is generally believed that displacement washing or lixiviation is the governing process. Cane diffusers have been adapted from the vegetable oil extraction industry via the beet industry. A considerable portion of Chemical Engineering solid-liquid extraction design theory has been developed in the vegetable oil industry where it is generally accepted that molecular diffusion from within to the surface of the solid material is the rate controlling mechanism, followed by surface washing—eddy diffusion. In the present paper the McCabe-Smith calculation is used to determine the number of ideal stages for an average diffuser on the basis of constant underflow. The stage efficiency is calculated from a knowledge of the diffusion coefficient and particle size distribution in the feed. These data are available from laboratory and pilot scale tests. From the number of ideal stages and stage efficiency, the number of actual stages is calculated giving results which are in close agreement with the number of stages used in existing cane and first bagasse diffusers. It is concluded therefore that the theory is applicable to cane diffuser design and that molecular diffusion within the cane particles is the controlling mechanism.

## 1. Introduction

Continuous countercurrent diffusion is well established in the beet sugar industry but is a relatively recent innovation in the cane sugar industry. Continuous diffusion of oil from vegetable materials has been practised since 1920, and the chemical engineering fundamentals for this unit operation have been expounded mainly by investigators in this industry.<sup>7,11,15,17,20</sup> Equipment for oil extraction has been adapted via the beet industry to the extraction of sucrose from cane—the majority of cane diffusers have almost identical counterparts in the oil extraction industry.

Procedures for the design of oil extractors on the basis of diffusion rates and corresponding stage efficiencies are well established and may be readily applied to many other fields. The delayed adaptation of diffusion equipment to the cane sugar industry may be attributed to the controversy over the mechanism controlling the extraction process. For example, Brüniche-Olsen<sup>4</sup> refers to the mechanism as “lixiviation rather than diffusion”, Payne<sup>21</sup> infers that “proper diffusion” occurs only across the intact cane cell wall and prefers the term “displacement washing”, while

Tantawi<sup>26</sup> agrees that it is a “lixiviation process where the part of *true* diffusion is limited only to the cells which escape rupturing”. Leaders in the adaptation of oil extractors and beet extractors to the cane sugar industry have therefore tended to ignore the importance of diffusion as a possible controlling mechanism in the extraction of sucrose from cane.

The present author has suggested that diffusion is the controlling mechanism in cane extraction in the absence of milling<sup>6</sup> and a recent investigation into the extraction of impurities by modern cane diffusers reported by Graham et al<sup>12</sup> has indicated a higher *rate* of extraction of invert sugar than of sucrose, corresponding to the ratio of the respective diffusion coefficients. This suggests that molecular diffusion controls the rate of extraction.

The purpose of this paper is to show that a cane diffuser can be designed by applying Chemical Engineering theory, firstly to predict the number of *ideal stages* required for a given extraction and secondly to predict—from known diffusion rates—the stage efficiency in practice. Finally, by a comparison of the predicted number of *actual stages* with the number required in existing diffusers the validity of the theory may be considered together with the proposed controlling mechanism of extraction—viz. molecular diffusion.

## 2. Proposed Extraction Mechanisms

### 2-1. Cane Milling

Sucrose extraction by conventional milling is a classic example of a washing process.<sup>25</sup> All the extracted sucrose is washed from opened cells and the material is finely ground during milling—60 per cent is under  $\frac{1}{8}$ " in final bagasse.<sup>5</sup> No attempt is made to saturate the bagasse evenly at each stage since the washing occurs during expression when the wash liquor is forced under high pressure through the partially compressed bed.<sup>6</sup> This turbulent displacement creates favourable mixing conditions. Since these conditions exist only momentarily, the rate of mass transfer is controlled by the liquid velocity and not by molecular diffusion.

### 2-2. Diffusion of Whole Beet and Cane Slices

A thorough study of beet diffusion has been made by Brüniche-Olsen.<sup>4</sup> Diffusion coefficients of  $1.7 \times 10^{-5}$  and  $1.0 \times 10^{-5}$  cm<sup>2</sup>/sec were reported respectively for free diffusion of sucrose in water and hindered diffusion through “killed” beet tissue—both at 75°C. At this temperature the cell membranes are permeable and hindered molecular diffusion controls the mass

transfer rate. In spite of this the present author in keeping with others<sup>24</sup> prefers the term "dialysis" since diffusion occurs mainly through a permeable membrane which restricts the diffusion of colloids. This definition gives a more precise distinction between diffusion across closed permeable cells (dialysis) and diffusion through a matrix of mechanically ruptured cells (hindered diffusion).

The diffusion of sugars from cane slices has been investigated by Zemek<sup>29</sup> and van der Pol and Young.<sup>24</sup> The latter reported diffusion coefficients similar to those for beet slices—above.

### 2-3. Diffusion of Shredded and/or Milled Cane

It is generally accepted that leaching involves molecular diffusion of solute through the porous solid phase followed by diffusion into the bulk liquid. The molecular diffusion rate may be hindered by the skeletal structure of the solid as in leaching natural materials and in this case the movement of the external fluid has little influence.<sup>28</sup> The present author conducted extensive tests on the leaching of shredded cane and bagasse—containing more than 90 per cent sucrose in *open* cells—which indicated that in spite of efficient external washing equilibrium was reached only after one hour.<sup>5</sup>

The reason for the slow leaching rate becomes clear after a microscopic examination of cane cells. In figs 1 (a) and (b) microscopic views of lateral and longitudinal cane sections are reproduced (see acknowledgements). Individual juice storage cells appear to measure in the region of 100 microns in diameter by 400 microns in length. Assuming that cell breakage facilitates lateral diffusion then displacement washing would necessitate percolation through

$0.5 \times 10^4 / 100 = 50$  cells for a 0.5 cm particle (see section 4-3). Since only from 0 to 5 ft of pressure head is available (in all cane diffusion operations) it is *unlikely* that displacement washing occurs *internally*. Hence as defined in the previous paragraph molecular diffusion probably controls the extraction rate.

It is perhaps anomalous to find that; while molecular diffusion is defined<sup>8,18,23,27</sup> as the transport of molecules through a fluid which is stagnant (or, if in laminar flow, at right angles to the streamlines) under the action of a concentration gradient; leading sugar technologists claim that *true* diffusion occurs only across a permeable membrane (see section 1).

The author is not aware of any comprehensive investigations into the mechanism controlling the diffusion of prepared cane. However, recent investigations at the Sugar Milling Research Institute suggest that molecular diffusion is controlling. Van der Pol and Young<sup>24</sup> determined diffusion coefficients averaging  $0.6 \times 10^{-5}$  cm<sup>2</sup>/sec for diffusion of shredded cane with 80 per cent available sucrose after tests on a pilot scale countercurrent diffuser processing about 500 lb/hr. Water was fed at 95°C. Unfortunately no particle size measurements were made and in order to calculate the above diffusion coefficient the mean thickness of particles was extrapolated from a relationship between available sucrose and thickness of cane slices. In the present author's opinion this inferred particle size is not strictly valid. Typical size distributions are presented in this paper (see appendix 3b and fig. 4) and from these data it appears that a more correct diffusion coefficient for the diffusion of cane during the tests by van der Pol and Young would have been 1.92 cm<sup>2</sup>/sec (see calculation in appendix 1). This is very close to the coefficient for free molecular

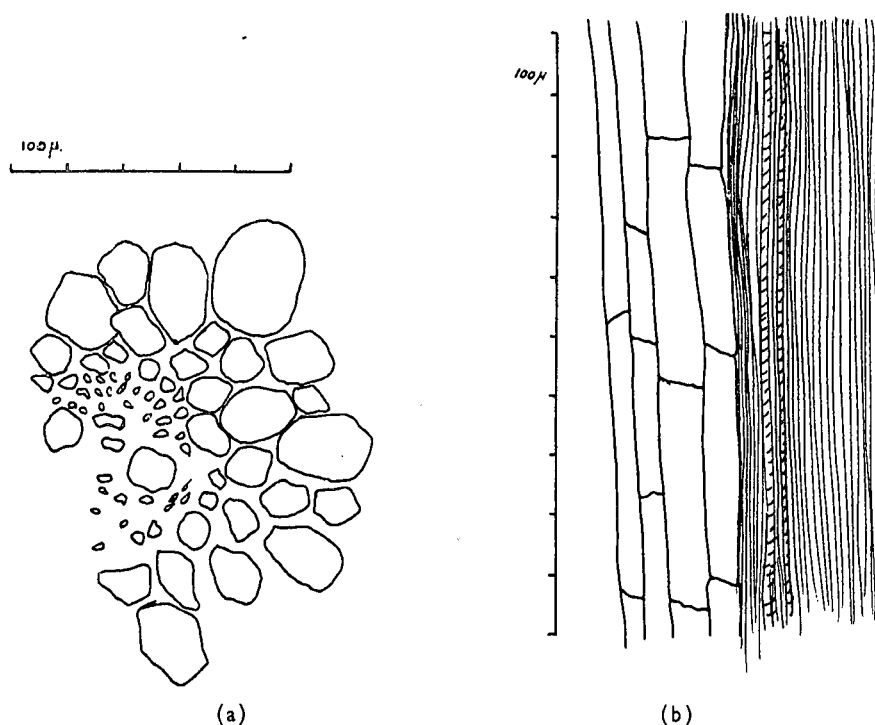


FIGURE 1: Microscopic view of section through cane (a) lateral (b) longitudinal —graduations in microns.

diffusion of sucrose in water at the actual diffusion temperature used in the tests and suggests that molecular diffusion is the controlling mechanism.

More recently the extraction of sugars and non-sugars has been investigated by Graham et al.<sup>12</sup> An analysis of their results from existing diffusers showed that the rate constants for the extraction of sucrose and invert sugars were in the same ratio as the corresponding diffusion coefficients. This also infers that molecular diffusion is the controlling mechanism.

It is evident from discussion in this section that by definition, from pure logic and from circumstantial analytical results there is a strong inference that molecular diffusion is the controlling mechanism in the diffusion of sucrose from prepared cane. Furthermore, since the diffusion coefficient for intact beet slices and cane slices are only slightly less than that for free sucrose in water at the same temperature, it appears that the coefficient for shredded or milled cane with 90 per cent opened cells should be very close to that of free sucrose. This appears to be confirmed by practical tests quoted above. In the following sections it will be shown that by the assumption of this mechanism fundamental Chemical Engineering theory can be applied to the design of a cane diffuser.

### 3. Determining the Number of Ideal Stages

#### 3-1. The Ideal Stage

An ideal stage in leaching is one in which complete mixing occurs between the washing (imbibition) liquid and the solution to be leached from the solid (fibre)—complete equilibrium is attained. In practical solid-liquid extraction this occurs only in surface washing operations<sup>19</sup> and stage efficiencies of about 40 per cent are more usual for leaching operations.

In the latter case mixing is dependent on the solid structure and diffusion rates.

#### 3-2. Constant Underflow

If the underflow (bagasse) after drainage at each stage contains the same solvent (water) to solid (fibre) ratio then the graphical determination of ideal stages<sup>6</sup> may be simplified and can be substituted by a simple calculation. While this is not the case in milling, constant underflow is likely to be closely approached in cane diffusion.

The retention of liquid in a wide variety of permeable solid beds has been investigated by Dombrowski et al.<sup>10</sup> who concluded that the drain height may be calculated from

$$Z = 0.275 / (P/g)^{0.5} (\rho/\sigma) \dots \dots \dots (1)$$

From the data in table 1, sucrose solutions over the concentration range found in diffusers (0° to 13° Brix) show a very small increase in  $\rho/\sigma$  ratio. It is probable that bagasse permeability decreases slightly due to compaction but since P is the power of 0.5 the effect on Z should be small. In fact Bruijn<sup>1</sup> has determined that after initial compaction during the first washing the bulk density remains constant. Hence it may be assumed that constant juice to fibre ratio is reached after each stage. (It should be mentioned that the subsequent calculations have been repeated assuming constant water to fibre ratios and almost identical conclusions were obtained).

#### 3-3. Calculation of Ideal Stages

In fig. 2 a simple diagrammatic explanation is given of the McCabe-Smith method<sup>16,19</sup> for calculating the number of ideal stages in the case of constant underflow. The leaching battery depicted in fig. 2(c) indicates that the first stage (stage 0) is omitted from

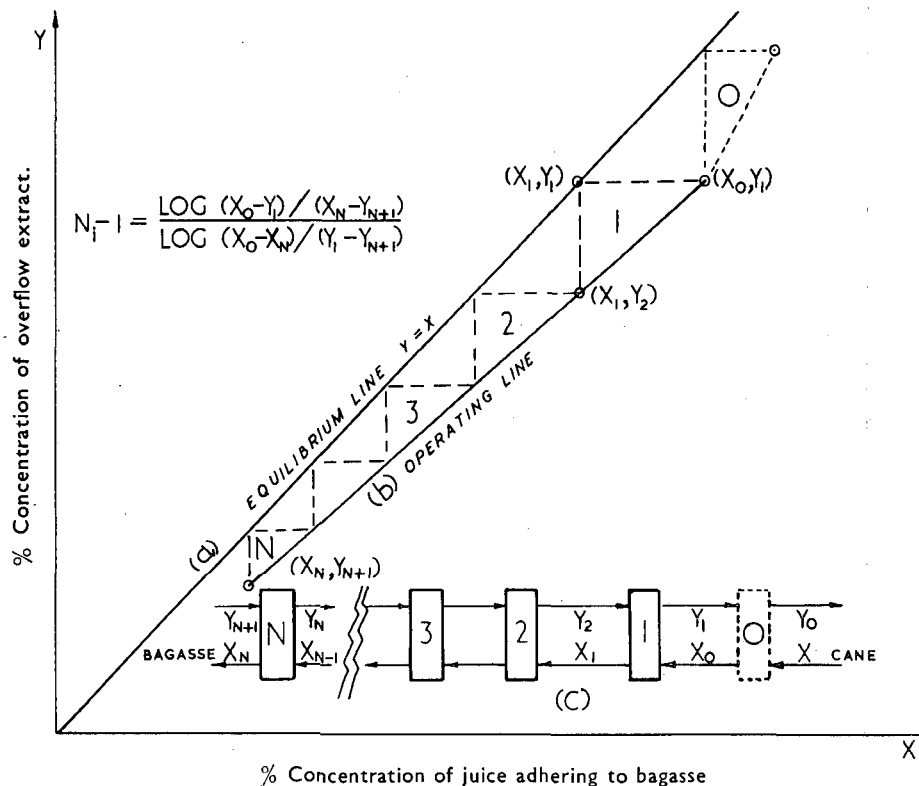


FIGURE 2: Determination of number of ideal stages by McCabe-Smith method for constant underflow.

the direct calculation. This is usually necessary since the solid feed rarely conforms to consistent underflow. Streams  $Y_1$  and  $X_0$  are therefore determined by mass balance. Streams  $Y_{N+1}$  and  $X_N$  for the  $N$ th stage are also determined by mass balance. These four concentrations are substituted into the equation

$$N_i - 1 = \frac{\log(X_0 - Y_1)/(X_N - Y_{N+1})}{\log(X_0 - X_N)/(Y_1 - Y_{N+1})} \dots \dots (2)$$

which permits calculation of the number of ideal stages required in addition to the initial zero stage. Hence no knowledge of the intermediate stage conditions is required.

The development of the equation<sup>16,19</sup> is too cumbersome for inclusion in this paper but the equivalent simple graphical construction in fig. 2 (a and b) will clarify the procedure. In fig. 2(c) it may be seen that since *ideal stages* are assumed the overflow solution from each stage will be in equilibrium with the solution adhering to the underflow, e.g.  $Y_1$  is in equilibrium with  $X_1$  in stage 1. The locus of all such points is represented by the line  $Y=X$  called the *equilibrium line*. The terminal concentrations represented by  $Y_1$ ,  $X_0$ ,  $Y_{N+1}$  and  $X_N$  may be determined by mass balance and by plotting the points  $(X_0, Y_1)$  and  $(X_N, Y_{N+1})$ , the *operating line* is located. The latter is a straight line in the case of constant underflow and conditions at each stage may be determined by completing the stepped construction shown in fig. 2 (a and b). Each step represents one ideal stage. Thus the number of ideal stages is found. Equation (3) provides the solution without resorting to graphical construction.

The dotted construction in fig. 2 (a, b) illustrates that the *feed cane* or bagasse condition deviates from constant underflow.

3-4. Application to Cane/Bagasse Diffusion

In appendix 2 the above procedure has been applied to typical cane (i) and first mill bagasse (ii) diffusion data.<sup>22</sup> The only data required are the cane or first mill bagasse analysis, the diffuser bagasse analysis and the imbibition rate. In appendix 2 the first step (a) and (b) is to calculate by balance the concentration of juice adhering to the fibre in diffuser bagasse which, for ideal stages is the same as that in final bagasse, i.e.  $X_N$ . The concentration of press-water plus imbibition is then calculated,  $Y_{N+1}$ , and it is assumed that this is applied prior to the last diffuser stage. Since the number of *ideal stages* is only about one third of the number of *actual stages* the previous assumption is justified.

The next steps (d) and (e) involve an appraisal of the feed solid data and then by means of an overall balance the concentration of the concentrated extract leaving the diffuser,  $Y_0$ , may be determined. Since  $X_0=Y_0$ , as shown in fig. 2(c), this also determines the concentration of the juice in bagasse entering stage 1. Finally, (f), the concentration of the extract leaving stage 1 is determined by a balance over stages 0 and 1 thus giving  $Y_1$ .

The four values are substituted into equation (2) to calculate  $N_i - 1$  and finally  $N_i$ , the number of *ideal stages* as in (h).

3-5. Results of Calculation

The above calculation shown in appendix 2 has indicated that diffusers operating under conditions which are found locally would require 5.54 ideal stages for a cane diffuser and 3.77 ideal stages for a bagasse diffuser for conditions of constant underflow.

4. Stage Efficiency

4-1. Controlling Mechanism

It has been pointed out in section 2-3 that, in common with many other leaching operations, the stage efficiency is reduced by diffusion from the cellular matrix in the cane or bagasse particles before surface washing occurs. Fortunately data are available to suggest that the controlling diffusion rate is close to that of sucrose. These data are based on extensive laboratory and pilot scale tests (see section 2-3).

With these data available the stage efficiency may be estimated by applying the theory below.

4-2. Calculation of Stage Efficiency

The mathematical development of the relationship between stage efficiency and diffusion coefficient is fairly complex. For the purpose of this paper it suffices to mention that the relationship is based on an analogy with the heating and cooling of an infinite slab of known thickness by unsteady-state conduction at both sides.<sup>19</sup> Equations developed for the latter operation are adapted to diffusion by substituting concentrations for temperatures. In its simplest form the equation for the overall stage efficiency is<sup>19</sup>

$$\eta = 1 - \phi(\beta) \dots \dots \dots (3)$$

$$\text{where } \beta = \frac{D \theta}{r^2} \dots \dots \dots (4)$$

For the sake of simplicity the function  $\phi$  is obtained from the graph in fig. 3 without elaborating on the actual equation. The validity of this relationship may be checked by the more discerning reader in the appropriate reference.<sup>19</sup>

4-3. Application to Cane Diffusion

The use of equation 3 requires a knowledge of the residence time at each stage, the diffusion coefficient and the mean particle thickness. Percolation tests have shown that a 5 min residence time in each stage is necessary to allow for interstage drainage.<sup>13</sup> The controlling molecular diffusion rate coefficient for sucrose in water may be extrapolated at the diffusion temperature of 75°C using a relationship derived by Brüniche Olsen.<sup>2</sup>

$$K_T = K(1.0250)^{T-t} \dots \dots \dots (5)$$

giving a value of  $1.62 \times 10^{-5}$  cm<sup>2</sup>/sec.

In order to assess the value of  $r$ , particle size distributions by weight have been obtained from measurements on first mill bagasse at diffusion factories<sup>2,5</sup> (see acknowledgements). These are shown in table 2 and are plotted in fig. 4 on log-probability graph paper. From the mean and standard deviation for the average plot—all show a normal distribution—the mean weight diameter may be determined. Alternatively, in the case of diffusion, the mean surface volume diameter would be more appropriate and having obtained the mean and standard deviations

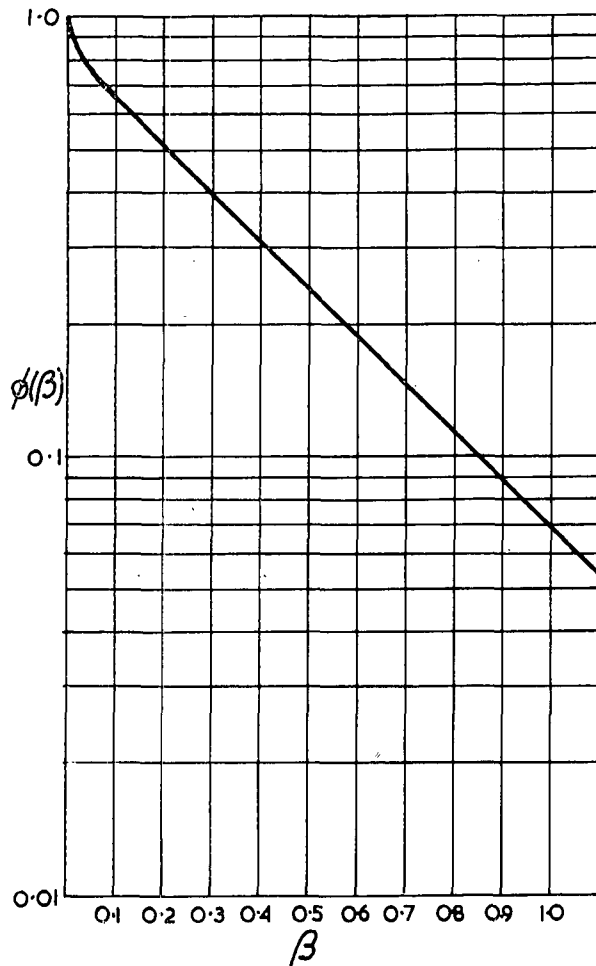


FIGURE 3: Graph <sup>3b</sup> for determining  $\phi(\beta)$

for the weight frequency plot the mean surface volume diameter may be calculated<sup>9</sup> from

$$\log d_{sv} = \log M - 1.151 \log^2 s \dots \dots \dots (6)$$

In appendix 3 it is shown that the mean diameter of first mill bagasse particles is 0.46 cm. Applying the above data to equation (4) the value of  $\beta$  is 0.092 as calculated in appendix 3.

From fig. 3 the value of  $\phi(\beta)$  is 0.66 and hence the stage efficiency is 0.34 or 34%.

4-4. Number of Actual Stages

Having established that the number of ideal stages is 5.54 for a cane diffuser and 3.77 for a first mill bagasse diffuser we may apply the equation

$$N_a = N_i / \eta \dots \dots \dots (7)$$

where  $\eta = 0.34$  and calculate that a cane diffuser would require 17 stages while a bagasse diffuser requires 11 stages (see appendix 3d).

*In fact existing cane diffusers have about 18 stages and bagasse diffusers have 10 or 11 stages. Hence the basic design has been achieved using chemical engineering principles with a minimum of operational data.*

5. Discussion

5-1. Controlling Mechanism in Cane Diffusion

Since it is possible to derive the number of stages required in a cane or bagasse diffuser accurately by (a) assuming that molecular diffusion controls the extraction process and (b) applying appropriate chemical engineering design theory it appears that the assumed controlling mechanism applies to diffusers in practice. If displacement washing was the controlling mechanism then (a) the stage efficiency should be higher than 34% and (b) an incorrect number of actual stages would be determined from theory.

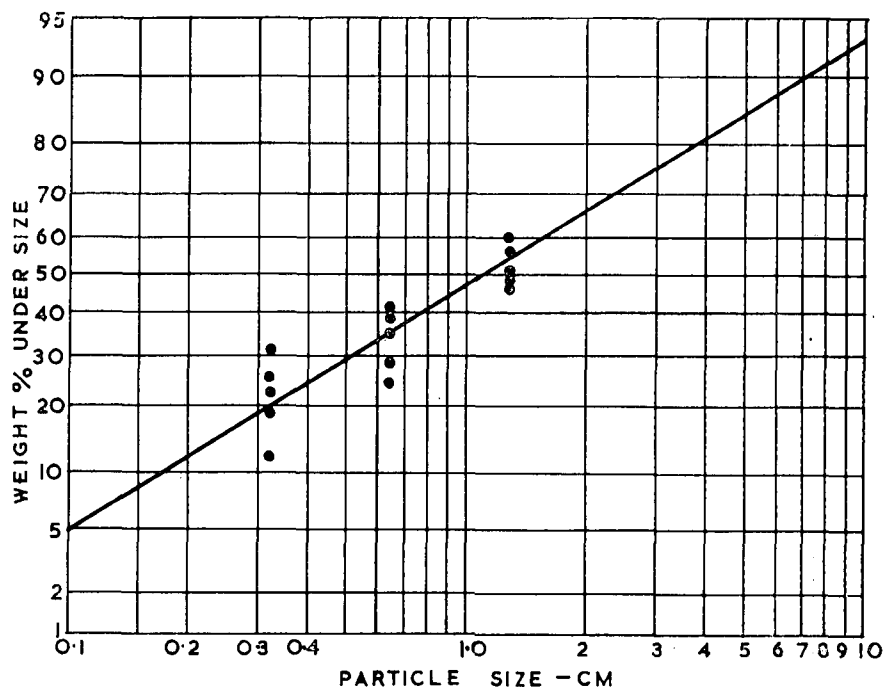


FIGURE 4: Mean particle size distribution for first mill bagasse.

It could be argued that, due to channelling, the efficiency of displacement washing is reduced. However, the stage efficiency of 34% corresponds exactly to the diffusion coefficient for free sucrose in water under conditions found in a commercial diffuser. It is unlikely that the surface washing rate would coincide exactly with this diffusion rate unless the latter was controlling. Hence the only logical conclusion is that molecular diffusion is the controlling mechanism. It must also be concluded that, while the matrix of opened cells in the prepared cane particles is sufficient to prevent free percolation through the mean size particle, the hindrance to free molecular diffusion is negligible. This confirms the mechanism proposed in section 2-3 and already suggested by practical investigation.

A further point is that in the initial stages of cane diffusion displacement washing should (if controlling) be more efficient than in bagasse diffusion since the bagasse enters at a very much lower juice/fibre ratio than cane. More absorption would occur than displacement in the case of a bagasse feed. However, it is found in practice that (section 4-4) the stage efficiency for a cane diffuser is  $5.54/18=0.31$  compared to  $3.77/11=0.34$  for a bagasse diffuser. This also indicates that displacement washing is not the controlling mechanism.

#### 5-2. Application to Diffuser Design

Since the basic design of a diffuser for bagasse or cane can be predicted by the application of standard Chemical Engineering theory the validity of this theory in cane extraction problems has been proven. The fact that molecular diffusion is the controlling mechanism is an important one. On this basis diffusion coefficients determined for various types of prepared cane may be applied to calculate the effect of cane preparation on the performance of a commercial diffuser.

The rate of extraction of certain soluble impurities may also be predicted from their diffusion coefficients and from equation (4) the importance of particle size may be seen. Furthermore the optimisation of stage retention time may be facilitated using this equation.

### 6. Conclusions

It has been shown that Chemical Engineering theory may be applied successfully to the design of a cane diffuser. On the assumption that molecular diffusion controls the sucrose extraction rate, designs corresponding to existing commercial diffusers may be predicted. This has confirmed the assumed mechanism.

#### Nomenclature

D	= diffusivity of sucrose through cane particles, $\text{cm}^2/\text{sec}$
$d_{sv}$	= mean surface to volume size, cm
K	= diffusion coefficient $\text{cm}^2/\text{sec}$ ; $K_T$ at higher temperature and $K_l$ at lower
M	= mean weight diameter, cm
N	= number of stages; $N_a$ , actual stages and $N_i$ , ideal stages
P	= permeability of porous bed, $\text{cu ft}/\text{hr}^2$
r	= length of diffusion—half minimum particle dimension, cm

s	= standard deviation of particle size distribution by weight
X	= concentration of solute in underflow solution; $X_0$ , leaving stage 0; $X_1$ , leaving stage 1; $X_N$ , last diffuser bagasse
Y	= concentration of solute in overflow solution; $Y_0$ , in extract; $Y_1$ , entering stage 1; $Y_{N+1}$ , in imbibition
Z	= drain height or height for average saturation in drained bed, ft
$\beta$	= dimensionless number characteristic of diffusion conditions and equivalent to Fourier number in heat transfer
$\theta$	= retention time at each diffusion stage, sec
$\phi$	= function of $\beta$ dependent on concentrations and obtained from fig. 3
$\rho$	= density of liquid, $\text{lb}/\text{cu ft}$
$\sigma$	= surface tension of liquid, $\text{lb}/\text{ft}$
$\eta$	= stage efficiency

#### Acknowledgements

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### Appendix

#### 1. Diffusion Coefficient by van der Pol and Young<sup>24</sup>

The authors found from performance data on the pilot scale diffuser that  $K/r^2 = 3.076 \times 10^{-4} \text{ sec}^{-1}$ . In order to evaluate K, the value of r was inferred from a relationship between available sucrose and thickness of solid cane slices. Thus for 80 per cent available sucrose a thickness of 0.14 cm solid cane slice was determined. The value of K was calculated as  $K = 3.016 \times 10^{-4} \times (0.14)^2 = 0.60 \times 10^{-5} \text{ cm}^2/\text{sec}$ . (The authors mentioned that this was a tentative procedure which should be checked by further investigation of the relevant particle size). The latter figure is too low to warrant serious consideration—particularly in view of the results in section 2-2.

In the present author's opinion this inference of particle size is not strictly valid since the true diffusion path length is considerably greater than 0.14 cm in the case of shredded cane<sup>5</sup>. A length of 0.5 cm (section 3b) would be appropriate and K would then be  $3.076 \times 10^{-4} \times (0.25)^2 = 1.92 \text{ cm}^2/\text{sec}$ . This is very close to the coefficient for free molecular diffusion of sucrose in water at the diffusion temperature and suggests that molecular diffusion is the controlling mechanism.

#### 2. Calculation of Ideal Stages

For the purpose of illustration the calculation may be simplified by considering the system: sucrose, natural fibre and water without error in the final result. Constant solution to underflow solid is assumed (see section 3-2) and typical cane and bagasse analyses are used as the basis<sup>22</sup>. Two cases are considered; diffusion of (i) cane and (ii) first mill bagasse.

##### (a) Final bagasse analysis

Assuming that sucrose % dry fibre is 4.50 and dry fibre % final bagasse is 45 then;

$$\begin{aligned} \text{Natural fibre \% bagasse} &= 4.5 \times 1.25 = 56.25 \\ \text{Juice \% bagasse} &= 100 - 56.25 = 43.75 \\ \text{Juice \% natural fibre} &= 4375/56.25 = 77.78 \\ \text{Sucrose \% natural fibre} &= 4.5/1.25 = 3.60 \\ \text{Sucrose \% juice} &= 360/77.78 = 4.629 \\ \hline X_N &= 4.629 \end{aligned}$$

##### (b) Diffuser bagasse analysis

Moisture % diffuser bagasse is usually about 85 and this corresponds to a natural fibre content of about 12.5 % hence:

$$\begin{aligned} \text{Juice \% bagasse} &= 100 - 12.5 = 87.5 \\ \text{Juice \% natural fibre} &= 8750/12.5 = 700.0 \end{aligned}$$

##### (c) Press-water and imbibition

Assuming ideal stages then juice in diffuser bagasse is homogeneous; thus juice expressed by dewatering mill:

$$\begin{aligned} \text{Juice \% natural fibre} &= 700.0 - 77.78 = 622.2 \\ \text{Sucrose \% juice} &= 4.629 \end{aligned}$$

Assuming that the imbibition rate on dry fibre is 250 % and that press-water and imbibition are combined for application prior to the last stage then:

$$\begin{aligned} \text{Imbibition \% natural fibre} &= 250/1.25 = 200 \\ \text{Imbibition juice \% natural fibre} &= 200 + 622 = 822.2 \\ \text{Sucrose \% imbibition juice} &= 4.629 \times 622.2/822.2 = 3.503 \end{aligned}$$

$$\hline Y_{N+1} = 3.503$$

##### (d) Analysis of Feed Solid

(i) Cane diffusion: Assuming 15 % dry fibre and 14 % sucrose in cane then:

$$\begin{aligned} \text{Natural fibre \% cane} &= 15 \times 1.25 = 18.75 \\ \text{Juice \% cane} &= 100 - 18.75 = 81.25 \\ \text{Juice \% natural fibre} &= 8125/18.75 = 433.3 \\ \text{Sucrose \% natural fibre} &= 1400/18.75 = 74.67 \\ \text{Sucrose \% juice} &= 7467/433.3 = 17.23 \end{aligned}$$

(ii) First bagasse diffusion: Assuming that the first mill achieves 60 % extraction then first bagasse has:

$$\begin{aligned} \text{Juice \% natural fibre} &= 433.3 \times 0.40 = 173.32 \\ \text{Sucrose \% juice} &= 17.23 \\ \text{Sucrose \% natural fibre} &= 1.733 \times 17.23 = 29.86 \end{aligned}$$

##### (e) Overall balance

(i) Cane diffusion:

$$\begin{aligned} \text{Juice in extract \% natural fibre} &= 200 + 433.3 - 77.78 \\ &= 555.5 \\ \text{Sucrose in extract \% natural fibre} &= 74.67 - 3.60 = 71.07 \\ \text{Sucrose \% juice in extract} &= 7107/555.5 = 12.794 \\ \hline Y_0 = X_0 &= 12.794 \end{aligned}$$

(ii) *First bagasse diffusion:*

$$\text{Juice in extract \% natural fibre} = 200 + 173.32 - 77.78 = 295.54$$

$$\text{Sucrose in extract \% natural fibre} = 29.86 - 3.60 = 26.26$$

$$\text{Sucrose \% juice in extract} = 2626/295.5 = 8.886$$

$$Y_0 = X_0 = 8.886$$

(f) *Balance on first two units:*(i) *Cane diffusion:*

$$\text{Juice \% natural fibre in underflow ex unit 0} = 700$$

$$\text{Sucrose \% juice in underflow ex unit} = 12.79$$

$$\text{Sucrose \% natural fibre in underflow ex unit 0} = 7 \times 12.79 = 89.53$$

$$\text{By balance: } 71.07 + 89.53 - 74.67 = 85.93$$

Juice entering unit 0 in extract \% natural fibre is equal to that applied as total imbibition = 822.2 hence:

$$\text{Sucrose \% juice in overflow from unit 1} = 8593/822.2 = 10.452$$

$$Y_1 = 10.452$$

(ii) *First bagasse diffusion:*

$$\text{Juice \% natural fibre in underflow ex unit 0} = 700$$

$$\text{Sucrose \% juice in underflow ex unit 0} = 8.886$$

$$\text{Sucrose \% natural fibre in underflow ex unit 1} = 7 \times 8.886 = 62.20$$

$$\text{By balance: } 26.26 + 62.20 - 29.86 = 58.60$$

Juice entering unit 0 in extract \% natural fibre is equal to that applied as total imbibition = 822.2 hence:

$$\text{Sucrose \% juice in overflow from unit 1} = 5860/822.2 = 7.127$$

$$Y_1 = 7.127$$

(g) *Terminal conditions:*(i) *Cane diffusion:*

$$X_0 = 12.794$$

$$Y_1 = 10.452$$

$$X_N = 4.629$$

$$Y_{N+1} = 3.503$$

(ii) *First bagasse diffusion:*

$$X_0 = 8.886$$

$$Y_1 = 7.127$$

$$X_N = 4.629$$

$$Y_{N+1} = 3.503$$

(h) *Calculation using equation (3)*(i) *Cane diffusion:*

$$-1 = \frac{\log(12.794 - 10.452)/(4.629 - 3.503)}{\log(12.794 - 4.629)/(10.452 - 3.503)}$$

$$= \log 2.080 / \log 1.175$$

$$= 0.3181 / 0.0701$$

$$= 4.54$$

$$N_i = 4.54 + 1 = 5.54 \text{ ideal stages}$$

(ii) *First bagasse diffusion:*

$$N_i - 1 = \frac{\log(8.886 - 7.127)/(4.629 - 3.503)}{\log(8.886 - 4.629)/(7.127 - 3.503)}$$

$$= \log 1.567 / \log 1.176$$

$$= 0.1951 / 0.0705$$

$$= 2.77$$

$$N_i = 2.77 + 1 = 3.77 \text{ ideal stages}$$

3. *Calculation of Stage Efficiency*(a) *Diffusion rate:*

Brüniche-Olsen gives  $0.54 \times 10^{-5}$  cm<sup>2</sup>/sec for the diffusion coefficient of sucrose in water at 23°C.

Using equation (5) the coefficient at 75°C is

$$K = 0.45 \times 10^{-5} (1.025)^{52}$$

$$= 1.62 \times 10^{-5} \text{ cm}^2/\text{sec}$$

(b) *Mean particle size:*

The data in table 2 are plotted in fig. 4 on log-probability graph paper. The mean, corresponding to 50% is 1.17. The ordinate value of 84.13 corresponds to a value on the abscissa of 4.6. The standard deviation is

$$s = 4.6/1.17 = 3.93$$

and  $M = 1.17$

Substituting into equation (6):

$$\log d_{sv} = \log 1.17 - 1.151 \log^2 3.93$$

$$d_{sv} = 0.4585 \text{ cm}$$

Note that the figures above apply to the mean plot though the points in fig. 4.

(c) *Stage efficiency:*

Since the mean diameter is 0.46 cm the mean diffusion distance is 0.23 (see section 2-3) and as discussed in section 4-3 the residence time for each stage is 5 min (or 300 sec). The latter may also be derived from measured permeability rates, viz. 1 cu ft/sq ft—min over a height of 5 ft requires 5 min for a complete displacement. The diffusion coefficient is  $1.62 \times 10^{-5}$  cm<sup>2</sup>/sec as shown in (a) above.

Using equation (4):

$$\beta = \frac{1.62 \times 10^{-5} \times 300}{(0.23)^2} = 0.092$$

From fig. 3  $\phi(\beta) = 0.66$  and from equation (3):

$$\eta = 1 - 0.66 = 0.34$$

The overall stage efficiency under the control of molecular diffusion is 34%.

(d) *Number of actual stages:*

Using equation (7) and data from appendix 2(h):

$$\text{No. of actual stages for cane diffuser} = 5.54/0.34 = 16.3$$

$$\text{No. of actual stages for bagasse diffuser} = 3.77/0.34 = 11.1$$

Table 1—Properties of Sucrose Solutions\*

<sup>o</sup> Bx	$\rho$	$\sigma$	$\rho/\sigma$
0	1	72.68	0.01376
6.8	1.0251	73.13	0.01402
10.0	1.0381	73.35	0.01416
13.1	1.0511	73.57	0.01429

\*Honig p4 and 54

Table 2—Particle Size Distributions For First Bagasse, Cumulative Weight Per Cent\*

Factory	>1.27cm	<1.27cm	<0.635cm	<0.318cm
EM	49.7	50.3	33.9	22.6
EN	43.1	56.9	37.8	25.6
UC	51.4	48.6	27.6	18.5
ML	40.0	60.0	41.6	30.8
UK	53.8	46.2	24.3	11.8

\*see acknowledgements

### Discussion

**Mr. Oosthuizen:** During the delivery of your paper you mentioned that since the equilibrium and operating lines were nearly parallel the overall stage efficiency was equal to the Murphree stage efficiency and hence equation (3) could be used for calculation of the former. Can this be proved mathematically?

**Mr. Buchanan:** Yes, this is correct, the mathematical background is discussed in Perry's Chemical Engineers' Handbook, 3rd edition, on page 551.

**Mr. Bax:** Do you consider in the case of a diffuser in practice that the recycle of the scalding juice through the cane bed is an actual stage?

**Mr. Buchanan:** Yes, in the data presented in this paper the scalding juice recycle stage has been added as an additional stage.

**Mr. Martin:** The stage efficiency has been calculated on the basis of the particle size of the prepared bagasse entering the diffuser. Since further disintegration occurs during the process to final bagasse surely this efficiency is incorrect.

**Mr. Buchanan:** The calculation refers only to the diffusion operation and the final mill is considered in a separate balance. No change occurs in particle size during actual diffusion hence it is correct to use the particle size of the diffuser feed material.

**Mr. Oosthuizen:** Do you have any proof that the operating and equilibrium lines are parallel? Have you conducted any separate tests to prove this?

**Mr. Buchanan:** If you examine the data in the appendix, e.g. section 2(g)(i) and (ii) you will see from the terminal conditions calculated by a simple mass balance on a typical diffuser that the ratio  $(X_0 - X_N)/(Y_1 - Y_{N+1})$  is very close to 0.86 and thus the operating line is nearly parallel to the equilibrium line. The overall stage efficiency will therefore be very close to the magnitude of the mean Murphree efficiency particularly as the efficiencies are fairly low.

**Mr. Jennings:** If these lines are parallel why are they not shown as such in fig. 2?

**Mr. Buchanan:** This is merely an illustration and is not intended to be quantitative. It simply illustrates a general case.

**Mr. Perk:** Do your calculations provide an indication of the relative attributes of cane diffusion as opposed to first bagasse diffusion.

**Mr. Buchanan:** This is primarily a question of economics; whether the cost of installing and operat-

ing the additional seven stages is less than that of the first mill with its auxiliary equipment. However, experience overseas with one particular type of diffuser indicates that cane can be prepared to a very fine degree without permeability difficulties in the case of a cane diffuser. Similar calculations to those in this paper have been carried out on the data from this particular diffuser and indicate that the stage efficiency is considerably higher with the result that the extraction is also higher. This would be expected from equation (4) which shows that the efficiency is related to the second power of the particle size. It seems possible that in bagasse diffusion the degree of preparation may be limited by the fact that the bed becomes less permeable after milling due to compression of the prepared material. If the costs to which I referred earlier are comparable then with a potential increase in extraction cane diffusion is an attractive alternative.

**Mr. Ashe:** Although your calculations can predict the number of stages required for a diffuser they do not determine the extraction which is all important from management aspect.

**Mr. Buchanan:** The purpose of this paper is to point out that, while it is the practice at present to build a diffuser and then to determine from tests what extraction can be achieved for a given throughput, by the application of a chemical engineering design procedure a required degree of extraction and throughput may be selected at the outset, and the basic diffuser design may be calculated without further tests. Although extraction has not been specifically mentioned the calculations in the appendix refer to usual conditions met with in practice of pol to fibre ratios in cane and bagasse. From these figures extraction may be calculated. However, I regard the pol/fibre ratio as being a more useful figure.

**Mr. Robinson:** From the previous discussion I get the impression that some of the questions infer a high degree of precision in these calculations concerning mass transfer. I believe that such calculations are only approximate.

**Mr. Buchanan:** The chemical engineer for obvious reasons does not usually attempt to work at the same level of accuracy as a physicist. However, the mass balances are accurate and the only approximations are those involved in the empirical determination of the stage efficiency. In spite of this the object of the paper has been to show that this empirical approach is justified by the fact that the predicted design is coincident with that in practice for a diffuser of the same duty.