FRICTION LOSS AND HEAT TRANSFER COEFFICIENT IN FINNED TUBE HEAT EXCHANGERS FOR REHEATING MASSECUITES

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Abstract

Equations developed for packed columns may be used to predict the overall heat transfer coefficient and friction loss in massecuite reheaters. The results seem to indicate that exchangers with the tubes staggered give higher heat transfer rates than those with the tubes in line.

Introduction

Although equations are available for predicting the friction loss across tube banks there are difficulties in applying them to banks of finned tubes because of the different fin types, pitches and sizes.

One solution to the problem may be to use the equations that have been developed for packed columns (Bird *et al*[2]). In this method the passages through the fins are regarded as a bundle of tangled channels of weird cross-section whose geometry can be described using a "mean hydraulic diameter", De being defined as follows:

$$\mathrm{De} = 4 \times \frac{\text{Volume of flow channels}}{\text{Wetted surface}} \qquad (1)$$

The actual velocity of the massecuite through these channels, V_a, is not of general interest but rather the superficial velocity V which is the average velocity that the massecuite would have had if no tubes were present. These two velocities are related by $V = V_a \xi$ where ξ is the "void fraction".

If we combine these definitions with the Hagen Poiseuille equation for pressure drop in viscous flow we have:

$$f = \frac{\Delta H . g . De}{2L \, . \, V^2} = \frac{16}{\xi(DeV\rho/\mu)} \qquad (2)$$

An assumption made in this equation is that the path of the fluid going through the tubes is of length L, the height of the tube bundle. Actually the massecuite goes through a tortuous path whose length may vary depending on the tube arrangement in the bundle, whether in line or staggered. The analysis of a great deal of data from packed columns (Bird *et al*[2]) has indicated that the length of the channels is 25/12 times the height of the column. In that case equation (2) becomes:

$$f = \frac{\Delta H . g . De}{2L \, . \, V^2} = \frac{100}{3\,\xi(DeV\,\rho/\mu)} \qquad (3)$$

Massecuite, however, is a pseudoplastic, non-Newtonian fluid and does not have a constant viscosity at a given temperature, but shows a decrease of viscosity with increasing shear rate. Its viscous properties can be represented over a limited range by the Ostwald-de Waele model or power law equation as it is usually called (Wilkinson[6]):

$$\tau = K\,(Sr)^n \qquad (4)$$

Where K, the consistency, is similar to the viscosity and n, the flow behaviour index, is a measure of the degree of non-Newtonian behaviour. The greater its departure from unity the more pronounced are the non-Newtonian properties of the massecuite.

The Reynolds number, $\frac{DeV\rho}{\mu}$, in equations (2) and (3) must then be replaced by its generalized form where it is expressed in terms of K and n, and instead we substitute:

$$Re = \frac{De^n . V^{2-n} . \rho}{K} \, . \, 8 \, . \left[\frac{n}{6n+2}\right]^n \qquad (5)$$

and the friction loss for non-Newtonians becomes:

$$\Delta\,H = \frac{32\;L\;V^2}{g\;De\;\xi\;(Re)} \qquad (6)$$

if we assume that the massecuite path is of length L, and

$$\Delta\,H = \frac{200}{3}\;\frac{L\;V^2}{g\;De\;\xi\;(Re)} \qquad (7)$$

if we assume that the massecuite path is of length 25L/12.

As for friction loss, it may be possible to apply packed column equations to predict the massecuite film heat transfer coefficient. An empyrical correlation (Yoshida *et al*[7]) recommended for viscous flow is:

$$Nu = 0{,}91\;(\mu/\mu_f)\;(Pr)_f^{\frac{1}{3}}\;(Re)^{\;0{,}49}\psi \qquad (8)$$

In this equation Nu, the Nusselt number, is defined as:

$$Nu = \frac{h_m De}{k} \qquad (9)$$

For massecuite, the generalized form of the Prandtl number, $Cp_m\mu/k$, for non-Newtonian power law fluids must be used. It is expressed as:

$$Pr = \frac{Cp_m K}{8k}\left[\frac{V}{De}\right]^{n-1}\left[\frac{6n+2}{n}\right]^n \qquad (10)$$

In equation (8) Pr is evaluated at the average film temperature.

The generalized form of the Reynolds number, Re, was defined by equation (5).

The shape factor, ψ, depends upon the shape of the packing used, and in the case of banks of finned tubes may depend upon the fin shapes and tube arrangement.

For non-Newtonian power law fluids equation (8) is written as:

$$Nu = 0{,}91\left(\frac{K}{K_f}\right)(Pr)_f^{\frac{1}{3}}\;(Re)^{\;0{,}49}\psi \qquad (11)$$

Experimental procedure

Measurements were taken on the massecuite reheaters at Illovo, Darnall, Umfolozi, Gledhow, Renishaw and Mount Edgecombe. Their geometrical characteristics are given in Table 1.

TABLE 1
Dimensions of Massecuite reheaters

		Mount Edgecombe	Reinishaw	Gledhow	Illovo	Darnall	Umfolozi
Heating area	m^2	1326	523,4	1500	535,8	1400	331,9
	m^2/T.C.H.	6,31	5,88	6,07	4,66	6,33	—
Length	m	8,128	3,048	9,14	5,1	7,112	4,077
Width	m	1,906	1,829	1,98	1,802	1,829	1,995
Sectional area	m^2	15,49	5,57	18,1	9,19	13,01	8,13
	m^2/T.C.H.	0,0738	0,0626	0,0733	0,0799	0,0589	—
Number of tube rows		8	12	10	10	14	6
Number of rows 25,4 mm pitch		4	8	6	10	10	6
Number of rows 38,1 mm pitch		3	4	4	—	2	—
Number of rows 50,8 mm pitch		1	—	—	—	2	—
Tube type (see Fig. 3)		*	B	B	A	C	B
Fins staggered		No	Yes	Yes	Yes	Yes	No
Tubes staggered		Yes	Yes	Yes	No	No	No
Height of bundle	m	1,493	1,518	1,263	1,13	1,663	0,72
Equivalent diameter	m	0,0631	0,05183	0,04778	0,06059	0,05239	0,05431
Void fraction		0,9042	0,801	0,786	0,8041	0,800	0,7899

First four rows type B. Next four rows 50 mm dia pipes with 240 mm fins
*

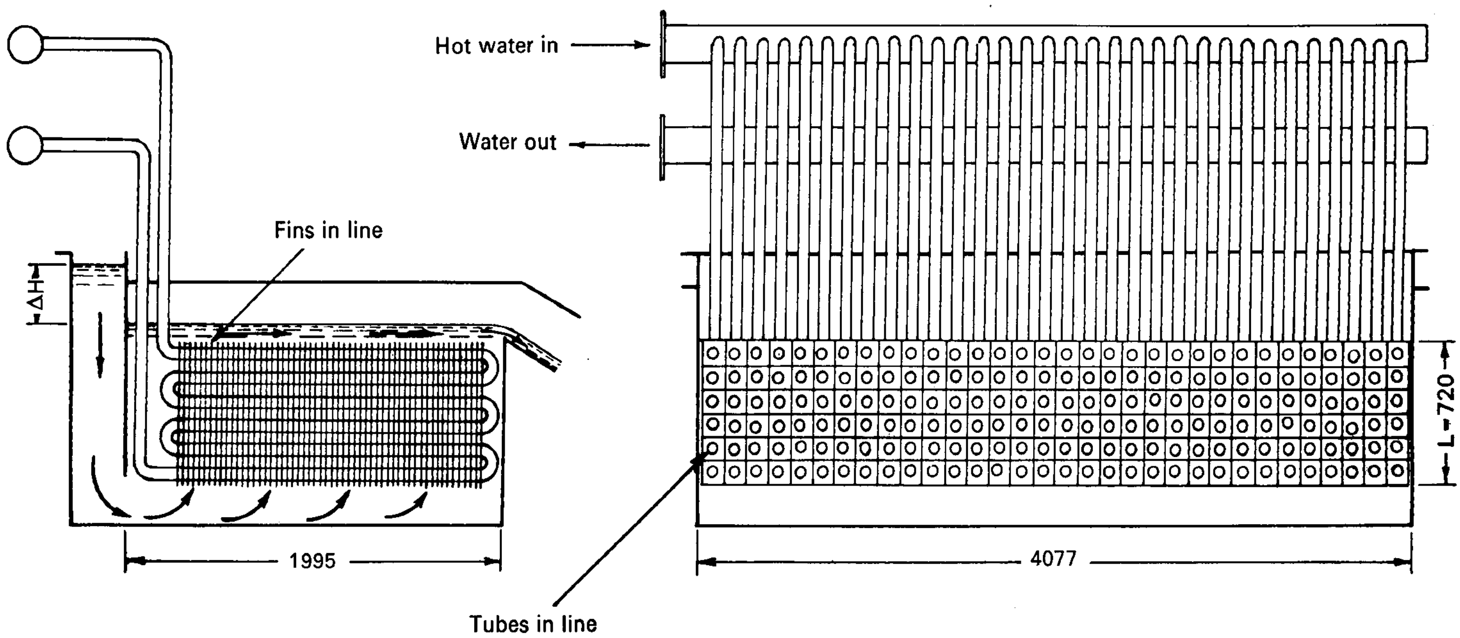


FIGURE 1 Umfolozi massecuite reheater.

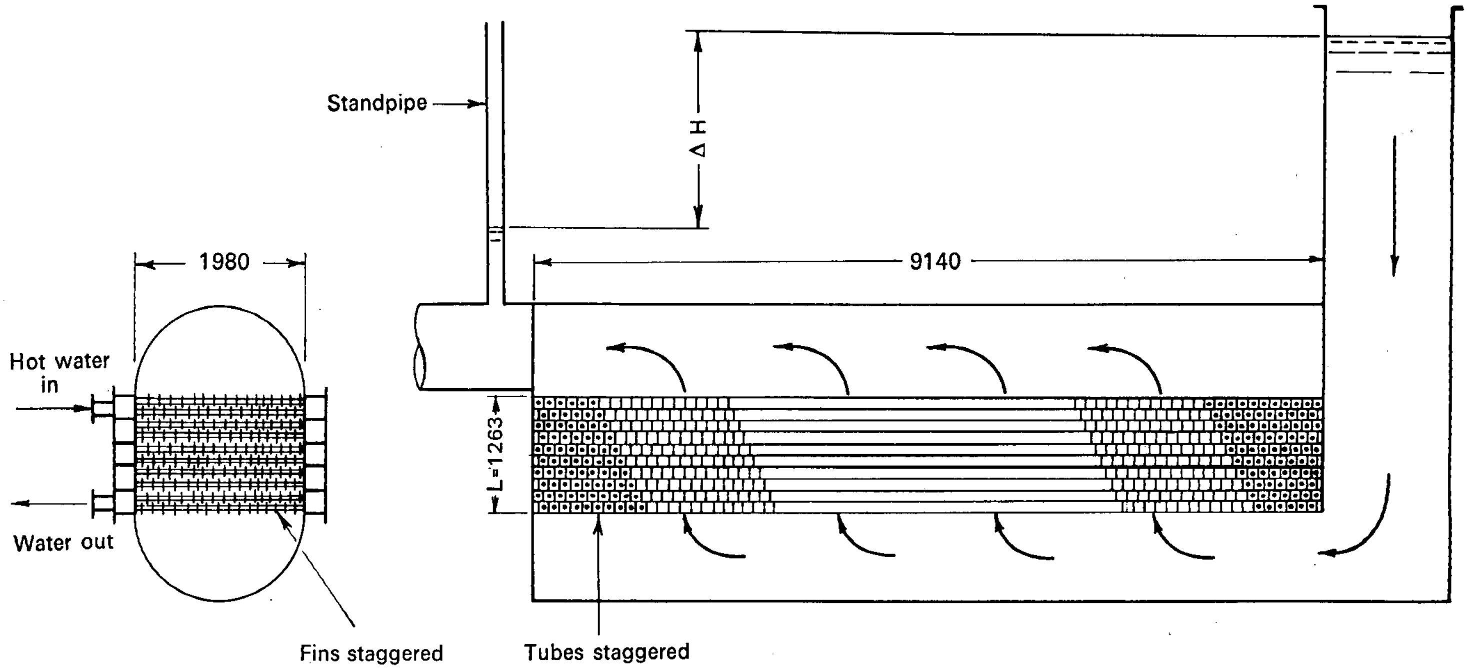
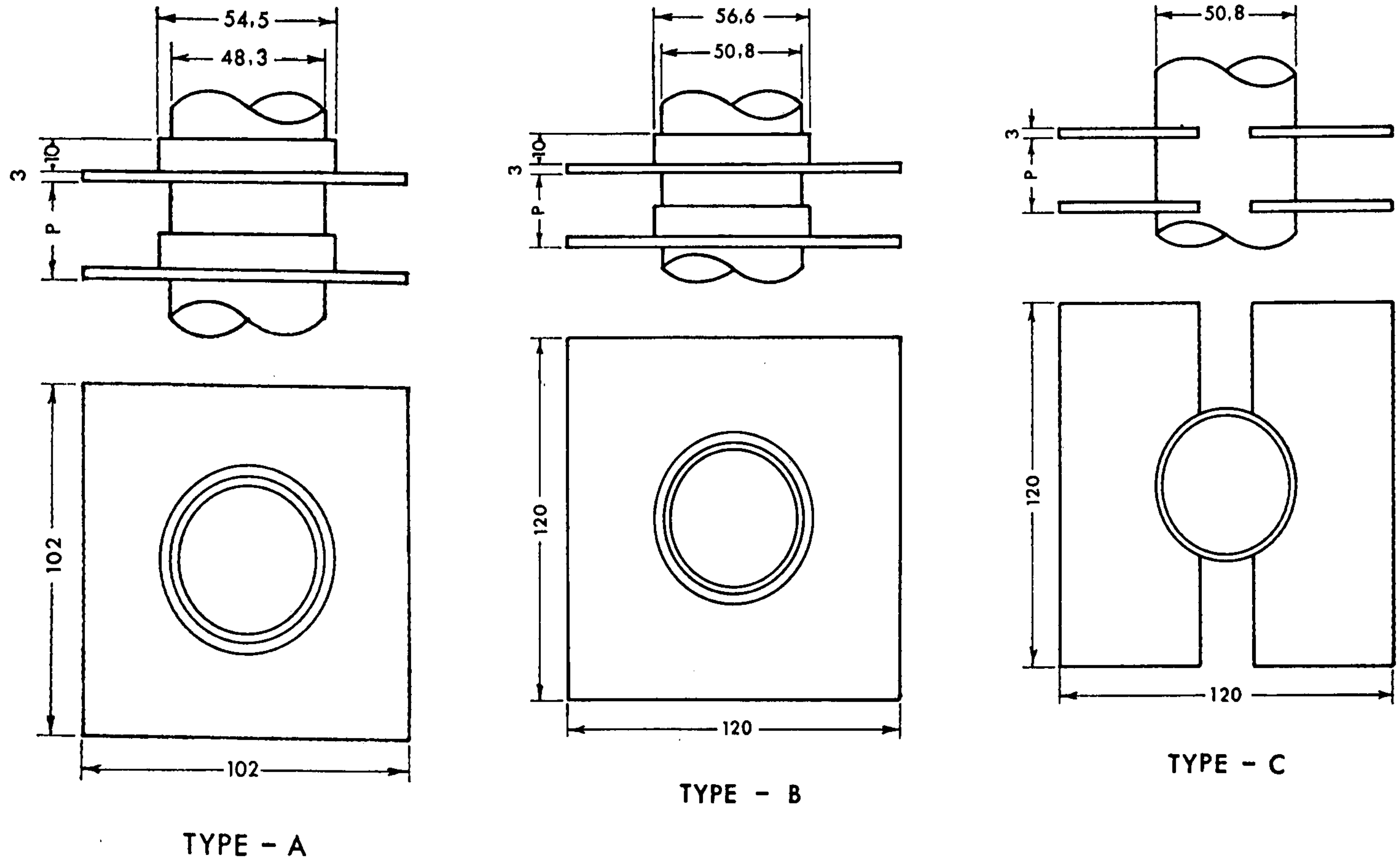


FIGURE 2 Gledhow massecuite reheater.



TYPE - A

TYPE - B

TYPE - C

FIGURE 3 Details of finned tubes.

Two of the reheaters, those of Illovo and Umfolozi, are of the overflow type as shown in Figure 1. Those of Darnall, Mount Edgecombe, Renishaw and Gledhow are of the totally enclosed type as shown in Figure 2.

The last three reheaters had a staggered tube arrangement and only Umfolozi did not have the fins staggered. The fin types are shown in Figure 3.

Neglecting heat losses, the rate of heat transfer in a reheater can be represented by:

$$G = W_w \, Cp_w \, (t_{w_i} - t_{w_o}) \qquad (11)$$

W_w, the mass rate of flow of the heating water, was obtained by measuring the flow rate with an orifice meter, and t_{w_i} and t_{w_o} were measured with mercury in glass thermometers accurate to 0,1°C. The inlet and outlet temperature of the massecuite was measured similarly. Some of the temperatures obtained are shown in Table 2.

As the rate of heat transfer can also be expressed as:

$$G = U.A.\ \Delta t_{\ell m} \qquad (12)$$

the overall heat transfer coefficient, U, was calculated from equations (11) and (12).

The overall resistance to heat flow, 1/U, is equal to the sum of the massecuite film resistance, the scale resistance, the tube wall resistance and the water film resistance, but the resistance of the massecuite film is so much greater that it was assumed that $U = \sim h_m$.

The mass rate of flow of the massecuite was obtained from a heat balance where:

$$W_m = \frac{W^w Cp^w \, (t_{w_i} - t_{w_o})}{Cp_m \, (t_{m_o} - t_{m_i})} \qquad (13)$$

in which the heat capacity of the massecuite, Cp_m can be expressed as a function of the brix (Hugot[4]).

$$Cp_m = (1 - 0{,}007 \, Bx)\, 4187 \qquad (14)$$

The superficial velocity of the massecuite was obtained from the expression

$$V = \frac{W_m}{\rho \, . \, S} \qquad (15)$$

in which ρ is the density of the massecuite as given in brix tables and S is the sectional area of the reheater.

The values used for the thermal conductivity are those given for the system sucrose-water (Honig[3]) and were extrapolated to cover the range from 90 to 100 brix.

The consistency, K, and the flow behaviour index, n, were determined by the method suggested by Skelland[1] using a Brookfield model HBT viscometer and spindle No. 7.

The flow behaviour index was obtained from the slope of a plot of the rotational speed of the viscometer versus the actual torque on logarithmic co-ordinates.

The shear stress was determined from the torque and dimensions of the spindle. As the spindle No. 7 is cylindrical, r is the radius and ℓ is the length.

$$\tau = \frac{\text{torque}}{2\pi \, r^2 \, \ell} \qquad (16)$$

The shear rate was calculated from the rotational speed of the viscometer and the flow behaviour index.

$$Sr = \frac{4\pi N}{n} \qquad (17)$$

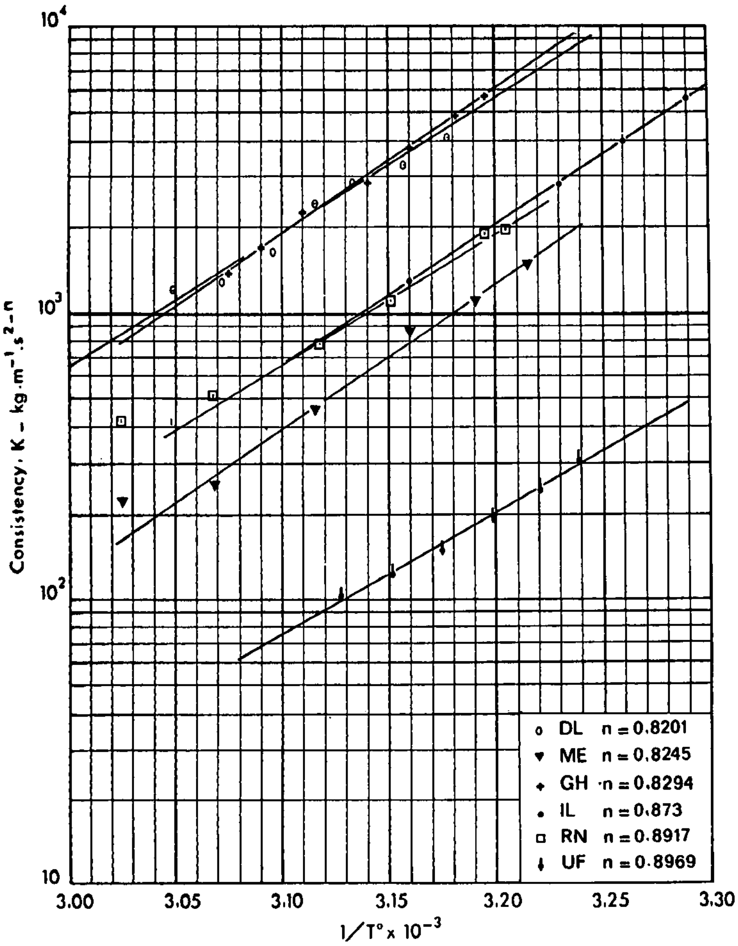


FIGURE 4 Rheological properties of "C" massecuites.

Substituting these values in equation (4) gave the consistency. This procedure was repeated at different temperatures to establish the consistency-temperature relationship. This can be expressed as:

$$K = a.10^{b/T_m} \quad (18)$$

where a and b are constants. Some of the consistencies obtained are shown in Figure 4.

The friction loss was obtained by measuring the difference in the level of the massecuite at the inlet and outlet of the reheaters some of the values obtained are shown in Table 2.

Results and discussion

Friction loss data

The results for the pressure drop studies are given in Figure 5 as a plot of the friction factor against the Reynolds number times the void fraction.

The results were regressed using a linear regression program and the relationship obtained was:

$$f = \frac{5,03}{\xi \; Re^{1,118}} \quad (19)$$

with a correlation coefficient of 0,956. On the same graph is shown the theoretical lines given by equations (2) and (3).

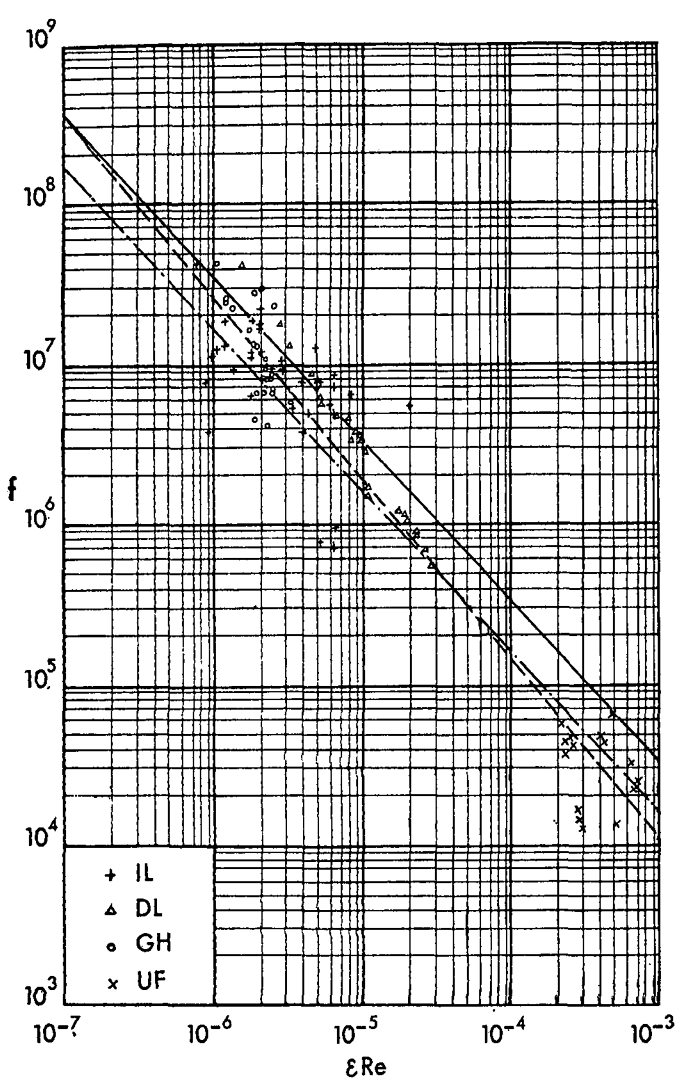
A slight divergence exists between them, but one must remember that this study was done under industrial conditions and that the following assumptions were made:

(1) In calculating the superficial velocity it was assumed that no channelling took place. This may not be the case in reheaters of the overflow type as by-passing may occur near the bends and return pipes. In addition the velocity was obtained from a heat balance in which the heat losses were neglected. The resulting error is proportionally greater at low massecuite flow rates.

TABLE 2

Data on temperatures, friction loss and overall heat transfer coefficient

	t_{w_i} (°C)	t_{w_o} (°C)	t_{m_o} (°C)	t_{m_i} (°C)	t_{w_i}—t_{m_o} (°C)	△H(m)	U(W/m²—°C)
Gledhow	57,0	56,1	55,2	39,9	1,8	1,72	6,87
	57,0	55,9	55,7	40,4	1,3	1,65	9,61
	57,0	56,23	54,2	40,3	2,8	1,73	5,10
	56,2	55,3	55,5	41,4	0,7	1,46	10,20
	56,5	55,3	55,2	41,6	1,3	1,43	11,40
	56,8	55,35	55,2	42,0	1,6	1,40	13,10
	56,5	55,5	55,3	42,0	1,2	1,41	9,85
	56,5	55,4	55,5	42,0	1,0	1,37	11,52
Illovo	52,0	51,6	43,5	36,8	8,5	0,43	3,14
	52,2	51,7	44,4	36,8	7,8	0,38	4,08
	52,3	51,9	47,13	36,8	5,17	0,34	3,87
	48,3	48,0	45,77	36,8	2,53	0,59	4,58
	48,6	48,0	44,97	36,7	3,63	0,53	5,22
	48,9	48,5	44,83	36,7	4,07	0,56	4,91
	49,3	48,9	44,83	36,6	4,47	0,58	4,61
	51,0	50,4	44,87	36,5	6,13	0,58	5,63
Darnall	60,6	59,7	54,6	49,3	6,0	3,09	7,50
	60,7	59,3	54,6	49,3	6,1	3,08	11,83
	60,8	59,6	54,7	49,3	6,1	3,17	9,98
	60,9	59,9	55,0	49,8	5,9	3,13	8,53
	60,9	59,9	54,6	50,4	6,3	3,10	8,53
	61,0	60,3	54,5	50,6	6,5	3,10	5,84
	61,0	60,2	54,6	50,8	6,4	3,07	6,83
	61,1	60,5	55,0	49,7	6,1	2,97	4,86
Umfolozi	64,4	63,9	50,5	44,5	13,9	0,02	8,75
	64,4	63,9	50,5	45,5	13,9	0,01	9,00
	64,4	63,9	51,0	45,5	13,4	0,01	9,15
	64,45	63,9	51,0	45,2	13,45	0,01	9,97
	64,35	63,85	51,7	45,5	12,65	0,025	9,51
	64,2	63,7	51,6	45,0	12,6	0,02	9,31
	64,15	63,6	51,5	45,0	12,65	0,025	10,32
	64,2	63,7	51,2	44,5	13,0	0,025	9,28



——— Theoretical line, Eq. 3

—·—·— Theoretical line, Eq. 2

— — — Experimental line, Eq.19

FIGURE 5 Pressure drop characteristics.

(2) It was assumed that the rheological properties of the massecuite did not change during each run. As some of the runs lasted for up to six hours it is probable that this was not the case.

(3) It was also assumed that the equivalent diameter was uniform throughout the tube bank. However, in reheaters of the overflow type about 20 per cent of the sectional area is taken up by the tube bends and return pipes and in reheaters of the totally enclosed type there are several rows of tubes with one pitch followed by several rows with another pitch.

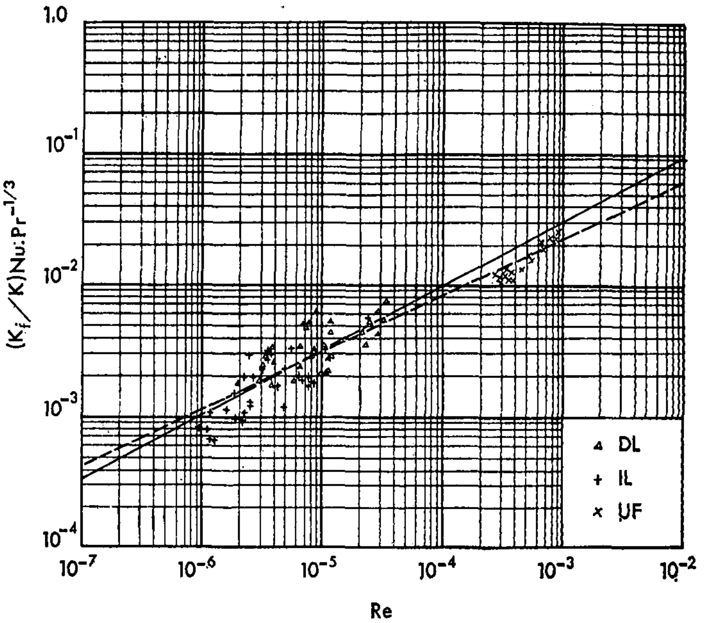
As can be seen from Figure 5, no difference can be observed between the friction loss with the tubes in line and the tubes staggered and also with the fins staggered. It seems from Figure 5 that the pressure drop lies approximately between the values predicted by equations (2) and (3).

We would suggest that equation (19) be used for evaluating the friction loss, where:

$$\Delta H = 10{,}06 \frac{L\ V^2}{g\ De\ \xi\ (Re)^{1{,}118}}$$

Heat transfer data

The heat transfer results are presented in Figure 6 as a plot of (K_f/K) Nu $(Pr)_f^{-\frac{1}{3}}$ versus the Reynolds number for reheaters with tubes in line.



——— Theoretical line, Eq. 11

— — — Experimental line, Eq. 20

FIGURE 6 Heat transfer characteristics for tubes in line.

These results were regressed using a linear regression program and the relationship was:

$$Nu = 0{,}44\ (K/K_f)\ (Pr)_f^{\frac{1}{3}}\ (Re)^{0{,}43} \qquad (20)$$

with a correlation coefficient of 0,931. On this graph is also shown the theoretical line represented by equation (11) which is slightly divergent. This divergence is probably the result of the assumptions mentioned previously. In addition, in evaluating the massecuite properties at the average film temperature, the tube wall temperature on the massecuite side was assumed to be the same as the average water temperature when of course there is a temperature drop through the water film and tube wall. This drop will increase as the massecuite film coefficient increases.

Although the reheaters tested had three different fin types, no difference can be observed in the results as shown in Figure 6. It must, therefore, be assumed that the shape factor is approximately the same for each type of fin.

In Figure 7 are shown the results for reheaters with staggered tubes.

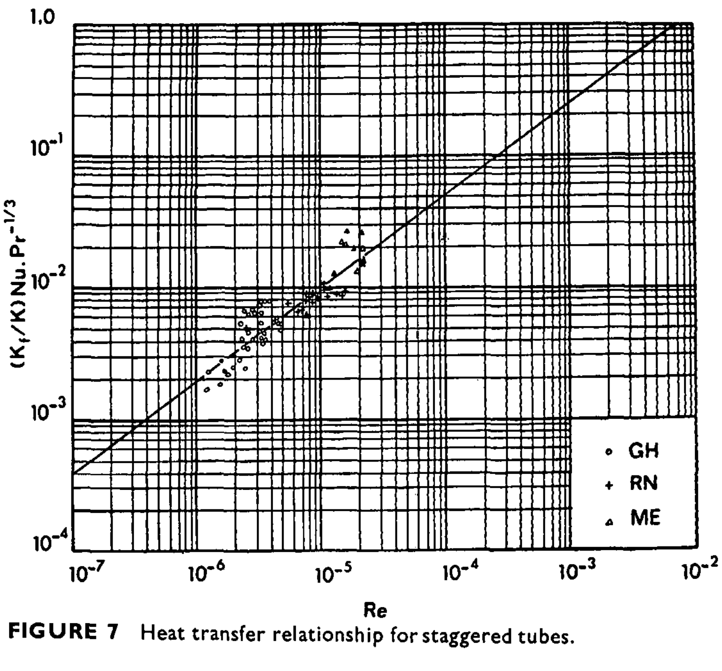


FIGURE 7 Heat transfer relationship for staggered tubes.

TABLE 3
Calculations of friction loss and overall heat transfer coefficients using equations (19) and (20)

ILLOVO: Brix of massecuite 96,0
Flow behaviour index 0,873

Tons cane/hour	Massecuite temperature		Water temperature		Heat Input	Overall heat transfer coefficient	Friction loss	Terminal temperature difference
	Inlet	Outlet	Inlet	Outlet				
100	35,0	45,0	52,30	51,97	23 508	3,83	0,64	7,30
110	35,0	45,0	53,06	52,70	25 851	3,94	0,69	8,06
120	35,0	45,0	53,82	53,43	28 214	4,04	0,74	8,82
130	35,0	45,0	54,58	54,16	30 557	4,13	0,78	9,58
140	35,0	45,0	55,34	54,89	32 920	4,21	0,83	10,34
150	35,0	45,0	56,10	55,62	35 263	4,28	0,87	11,10

DARNALL: Brix of massecuite 97,5
Flow behaviour index 0,8201

Tons cane/hour	Massecuite temperature		Water temperature		Heat Input	Overall heat transfer coefficient	Friction loss	Terminal temperature difference
	Inlet	Outlet	Inlet	Outlet				
221	35,0	55,0	63,08	62,17	97 944	4,44	3,23	8,08
221	38,0	55,0	61,11	60,34	83 252	4,75	2,75	6,11
221	41,0	55,0	59,51	58,87	68 560	5,05	2,35	4,51
221	44,0	55.0	58,19	57,69	53 869	5,34	2,01	3,19
221	47,0	55,0	57,10	56,73	39 177	5,62	1.72	2,10
221	50,0	55,0	56,19	55,96	24 486	5,90	1,47	1,19

The data on this graph can be represented by the following equation:

$$\mathrm{Nu} = 32{,}1\ (\mathrm{Pr})_f^{\frac{1}{3}}\ (\mathrm{Re})^{0{,}7} \qquad (21)$$

The correlation coefficient in this case is 0,9. In Figure 7 is also shown the experimental line for tubes in line. As can be seen the values of Nu $\mathrm{Pr}^{-\frac{1}{3}}$ are slightly higher than with the in-line tube configuration, which agrees with previous observations for heat transfer across staggered tube banks (Mc Adams[5]). Again no difference could be observed with different fin types.

Practical applications

In Table 3 are shown calculations of friction loss and overall heat transfer coefficients using equations (19) and (20).

In the example worked for the Illovo reheater the figures were calculated for crushing rates of from 100 to 150 tch using the average C massecuite volumes for 1974. It can be seen how the friction loss and terminal temperature difference increase with the massecuite throughput. The values for 100 tch compare well with the actual measurements given in Table 2.

The second example calculated using the Darnall reheater shows how the friction loss and terminal temperature difference decrease as the inlet temperature of the massecuite increases.

It illustrates the effect of decreasing the consistency. In this example the agreement between the measured and calculated values for an inlet temperature of 50°C does not seem to be good. The measurements, however, were taken at the end of the crushing season, and the rate of flow of massecuite observed was about 0,009 m^3/s whereas the calculated values are based on the average for the year which was 0,0024 m^3/s.

Acknowledgements

The author would like to thank the management and staff of Gledhow, Illovo, Darnall and Umfolozi for permission to perform these tests and for their co-operation in carrying them out.

Nomenclature

The symbols used in the text are listed below:

A = total heat transfer area
a = constant (eq. 18)
b = constant (eq. 18)
Cp = heat capacity $J.kg^{-1}.°C^{-1}$
De = mean hydraulic diameter m
f = Fanning's friction factor
G = rate of heat transfer
g = acceleration of gravity
ΔH = loss of head due to friction m
h = film heat transfer coefficient $W.m^{-2}.°C^{-1}$
K = power law consistency index $kg.m^{-1}.s^{n-2}$
k = thermal conductivity $W.m^{-1}.°C^{-1}$
L = length of flow of channel m
ℓ = length of viscometer spindle m
N = rotational speed of viscometer s^{-1}
Nu = Nusselt number
n = power law flow behaviour index
Pr = Prandtl number
R = rate of heat transfer $W.s^{-1}$
Re = Reynolds number
S = sectional area of reheater m^2
Sr = shear rate s^{-1}
T = absolute temperature °K
t = temperature °C
$\Delta t_{\ell m}$ = logarithmic mean temperature difference °C
U = overall heat transfer coefficient $W.m^{-2}.°C^{-1}$
V = superficial velocity of massecuite $m.s^{-1}$
V_a = average velocity of massecuite $m.s^{-1}$
W = mass rate of flow $k.s^{-1}$

Greek

ξ = void fraction
μ = viscosity $kg.m^{-1}.s^{-1}$
ρ = density $kg.m^{-3}$
τ = shear stress $kg.m^{-1}.s^{-2}$
ψ = shape factor

Subscripts

f = measured at average film temperature
i = inlet conditions
m = massecuite
o = outlet conditions
w = heating water

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