ANALYSIS OF WHEN TO PLOUGH OUT A SUGARCANE FIELD

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Abstract

A method for deciding whether a field should be ploughed out or ratooned once more, based on discounted cash flows, is developed in the following steps: (*a*) Correcting historical yield data to standard age and climatic conditions; (*b*) deriving a yield equation in terms of ratoon stage; (*c*) developing a method for calculating the average optimum replant cycle length for future replant cycles of a field; (*d*) deriving the plough-out threshold level of yield for a field. If the estimated yield of an additional ratoon from the existing root system is below this, the field should rather be ploughed out; (*e*) putting the entire procedure of correcting to standard yields, calculating plough-out threshold level and estimating yield of an additional ratoon crop on to computer for routine use on the Company estates.

1. Introduction

The decision when to plough out a ratoon crop of cane and replant is usually done when the yield has dropped "too far", without really specifying what this term means or on what basis it had been arrived at. Du Toit[2] and Boyce[1] provided a scientific basis for calculating at what stage a crop should be ploughed out and replanted. This paper shows a modification of the method, introducing the following refinements:

(*a*) The money expended on establishing the crop and the incomes received from selling the crop are discounted with respect to time, i.e. other things being equal, it would be more advantageous to put off the plough-out stage with its high costs by ratooning for another crop, because the value of money in the future, whether income or expenditure, is less than in the present time.

(*b*) An expression is derived for determining for any individual field the yield level below which ploughing out and replanting is preferable to ratooning once more.

(*c*) A distinction is made between the general dependence of the field performance on the ratoon stage, and the field performance during the present replant cycle, because the last mentioned performance might have been affected by special conditions which do not usually occur in its replant cycles.

(*d*) A system has been developed whereby a computer is used for performing the calculation whether or not to plough out a specific field.

2. Corrections to yield data

Because we want to maximise future income, we must try to predict the future performances of each field under review. Past performance of the field is relevant only in so far as it could provide a basis for predicting future performances, particularly in regard to how ratoon stage affects yield.

Among the more important factors which determine yield from a field (not necessarily in the order of importance) are:

Climatic conditions, i.e. where the crop had received favourable conditions of rainfall, sunshine, etc.

Age. Other things being equal, the older the crop, the higher the yield.

Ratoon stage.

Presence of diseases and pests.

Inherent field characteristics, especially soil type, depth of soil, aspect of the field, most of which are basically unalterable, and which determine whether a field is "good" or "poor".

When analysing past data to obtain the ratoon decline effect, it is necessary to try eliminating all other effects which would tend to obscure this. The data should therefore be compensated for seasonal climatic effects, which are random and non-repeatable, and for age of harvest.

2.1 *Source of data*

Since 1963 a large amount of data on the performances of the crops of Huletts estates at Darnall have been collected and stored on computer disc, for easy access for future processing by computer. Included in the relevant information for each crop are the tch yield, the age (from these 2 statistics the tons cane per hectare per month (tchm) can be calculated) and the ratoon stage. The ratoon stage is numerically represented by 0 for plant cane, 1 for first ratoon, 2 for second ratoon, etc.

2.2 *Climatic correction*

The reasonable assumption is made that the effect of climate during a given season on the yield from any field from a given estate will be proportional to the average tons cane/(hectare-month) (tchm) performance of all crops on that estate for that season.

For each season j of a specific estate e, a correction factor K_{ej} was calculated, namely,

$$K_{ej} = \frac{\text{Average tchm for all crops of estate e, all seasons}}{\text{Average tchm for all crops of estate e in season j}}$$

2.3 *Age correction*

To further make these tch yields comparable, they should be based on a standard age of harvest. Analysis of historical data showed that the average age of harvest for plant cane was 20 months, and for ratoon crops 19 months, and these were taken as the respective standard ages. For crops which had been harvested at other ages, one might be tempted to correct by an age ratio $K_A = \frac{\text{Standard age}}{\text{Actual age}}$ but statistical analysis in Appendix 1 shows that this over-compensates for non-standard age, and that a better correction is achieved by $\sqrt{K_A}$.

Thus for a crop harvested from estate e in season j,

$$\text{Standard tch} = \text{Recorded tch} \times K_{ej} \times \sqrt{K_A}$$

3. The yield equation

The next step is to express standard yields in terms of ratoon stage.

3.1 *Suggested yield equation*

The equation that most readily springs to mind is a simple straight line equation of the form

$$Y = A - Br \qquad (1)$$

where Y = Standard tch yield
r = Ratoon stage- r = 0 for plant cane,
r = 1 for 1st ratoon, etc.

Intercept A and slope B are parameters of the equation.

For plant cane, with r = 0, Y will be = A, and A will henceforth be referred to as the *initial yield parameter.*

B indicates by how much the yield drops with each succeeding ratoon stage, and will be called the *yield decline parameter.*

3.2 *Fitting of yield equation*

The average standard tch yields are plotted in Fig. 1 as a function of ratoon stage. These points do not form much of a straight line relationship, and beyond ratoon stage = 4, the yields level out and then start increasing again.

Linear regression analysis gave only a poor correlation of 0,26, and produced a low B-coefficient of 3,48, i.e. indicating that the yield decline was only about 3,5 tch with each successive ratoon stage.

However, these data could be biased through pre-selectivity. In practice a "poor" field would only be allowed a small number of ratoon crops before being ploughed out, whereas a "good" field would be ratooned many times, with the result that crops which correspond to the higher ratoon stages, represent only the high-yielding, better fields.

To eliminate this effect, all the data were sorted in classes of length of replant cycle, represented by n. For example, if a crop (say the second ratoon, r = 2), was part of a replant cycle which had lasted up to the fourth ratoon, then that crop would belong to the class of n = 4

The standard tch values, averaged per ratoon stage for each class of replant cycle length, are plotted in Fig. 2. This bears out the suspicion of bias, because:

The longer the length of replant cycle (i.e. the higher the value of n), the higher the yield for a given ratoon stage.

The ratoon decline lines within each replant cycle length class tend more towards looking like straight lines and have a significantly steeper slope than the common line which had been fitted in Fig. 1.

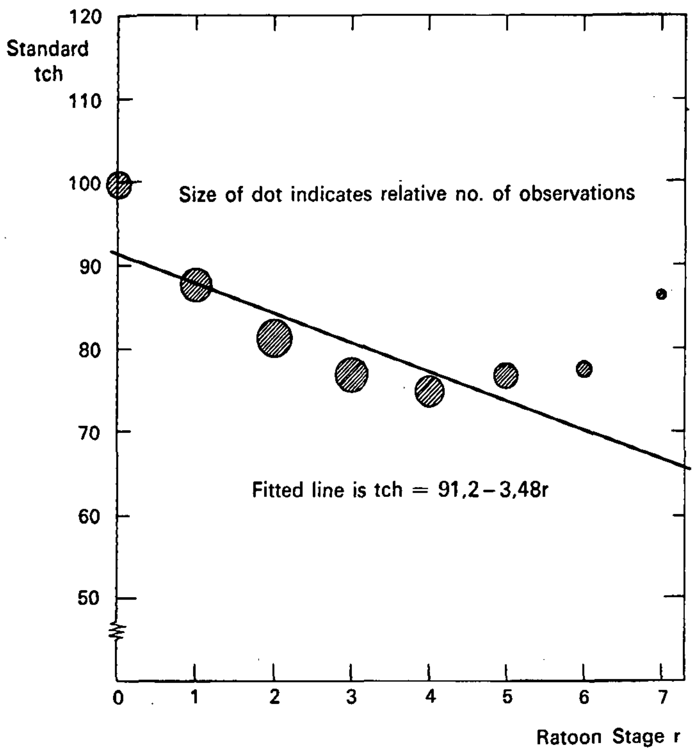

FIGURE 1 Average tch performances for Darnall, corrected to standard conditions for seasonal climate and age.

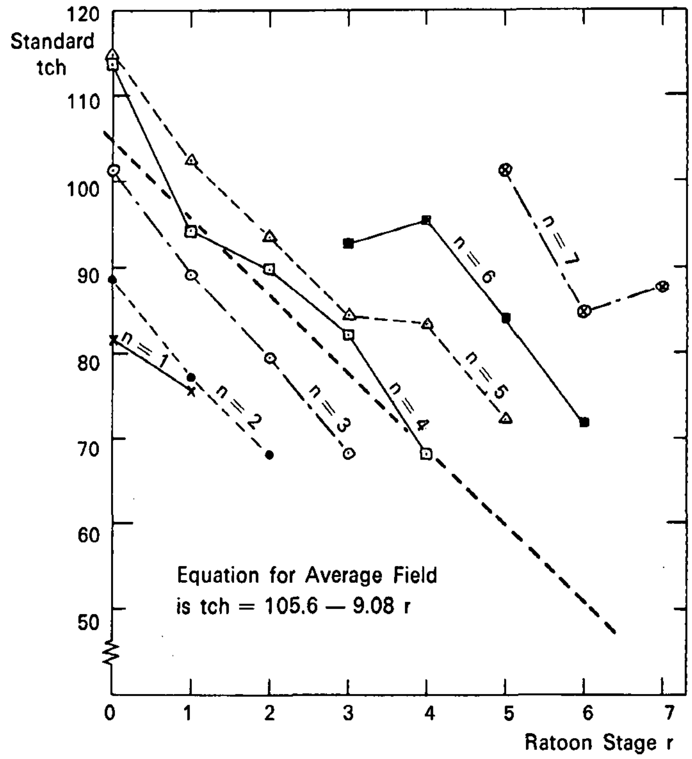

FIGURE 2 Average standard tch performances for Darnall, grouped per length n of replant cycle.

3.3 *Derivation of yield equation for an average field*

A linear regression analysis was done, where the dependent variable Y for each crop was standard tch, and the independent variables were the ratoon stage r of that crop, and the number n of ratoon stages in the replant cycle to which that particular crop belonged.

The following equation was obtained:

$$Y = 66{,}7 + 10{,}16n - 9{,}08r$$

with a multiple correlation coefficient of 0,51.

As expected, the rate of yield decline of 9,08 with increase in ratoon stage r is considerably higher than the 3,5 found above.

The average number of ratoons per replant cycle was 3,83, so that the yield equation for an average field becomes:

$$Y = 66{,}7 + (10{,}16 \times 3{,}83) - 9{,}08r$$
$$= 105{,}6 - 9{,}08r$$

For convenience this equation was rounded off to

$$Y = 105 - 9r,$$

which is the equation (1) with A = 105 and B = 9.

4. Data required

4.1 *Performance estimates*

The following expected or average future crop performance data for the successive ratoon stages of each *replanted* field, starting with a plant crop, are required:

Y_r = standard yield in tons cane per hectare (tch) of ratoon stage r for that field,

a_r = age (in years) of stage r at time of harvest.

For Darnall a_0 = 20 mo. = 1,67 years for plant crop (r = 0)
a_r = 19 mo. = 1,58 years for ratoon crops (r = 1, 2....)
f = fallow period after plough out (in years)

For Darnall f = 5 mo. = 0,41 years.

The estimated yield from the next crop from the *existing* root system of each field under review (say from ratoon stage r) is required:

y_r = standard yield in tons cane per hectare (tch) of ratoon stage r for that field.

4.2 *Cost data*

Ploughing out a field and replanting could be considered as making an investment with the object of achieving higher yields in the future. The problem therefore is one of deciding when to make the investment. Because the life of the "investment" can spread over as many as 10 years, the discounting effect of money should be taken into consideration.

If R = annual discount rate (as a fraction, not a %),
= 0,15 for this example,

$$\text{then discount factor } d = \frac{1}{1 + R} = 0{,}870$$

For a description of the principles of discounting, refer to Appendix 2.

To simplify the method and the calculations, all the cost and revenue cash flows generated during a given crop are forward discounted to the time of harvesting. For example, the cost of weeding and fertilizing a crop of ratoon stage r, which usually takes place at the beginning of that ratoon, must be forward discounted by a factor of $(1 + R)^{a_r}$, the exponent a_r (the age) representing the approximate elapse of time between incurring such costs and the time of harvest. Similarly, the cost of ploughing out and liming a field, which takes place at the start of the fallow period, should be forward discounted by a factor $(1 + R)^{(f + a_o)}$.

The cost and revenue items which follow were forward discounted where necessary. The values shown were chosen mainly for the sake of illustration, but are of a realistic order of magnitude.

C_H = cost per hectare of starting a ratoon crop. This includes items such as fertilizer, weeding, weedicides, applications of such chemicals
= R220/ha.

C_{HP} = cost per hectare of establishing a plant crop. This includes ploughing out, liming, replanting, seed cane, levelling, as well as fertilizer, weeding, etc.
= R850/ha.

C_T = cost per ton of cane when harvesting the crop. This covers costs which can more accurately be expressed per ton of cane than per hectare, and includes cutting, stacking, harvesting and infield transport costs
= R2/ton.

V_T = value per ton of cane obtained from the field
= R15/ton at average sucrose content.

5. Derivation of optimum number of ratoons

5.1 *Optimum average number of ratoons: General derivation*

Let P_r = profit per hectare of ratoon stage r based on average performance and with all costs and revenues discounted to the time of harvest of that crop:

$$P_r = V_T Y_r - (C_H + C_T Y_r)$$
$$= (V_T - C_T) Y_r - C_H \text{ for ratooned crops, } r = 1, 2 \ldots \quad (2)$$

$$P_o = (V_T - C_T) Y_o - C_{HP} \text{ for plant crop, } r = 0 \quad (3)$$

Except for where we get into excessively high values of r, with correspondingly low yield Y_r, P_o will usually be lower than P_r, because of the higher value C_{HP} for establishing a plant crop, compared with C_H for a ratoon crop.

Consider the time of ploughing out the root system of an old field as being the "present time", i.e. t = 0. The rth ratoon will be harvested a period of $f + a_o + a_1 + .. + a_r$ years in the future, generating a profit P_r Rand per hectare.

If, for convenience, we define the harvest date of the rth stage of the present replant cycle, referred to the present time, as $t_r = f + a_o + a_1 \ldots . + a_r$, the present value of the profit is discounted to $P_r d^{t_r}$.

If the average duration of a complete replant cycle is n ratoons for this field, the total present value of the successive discounted profits is given by:

$$U_n = P_o d^{t_o} + P_1 d^{t_1} + \ldots . + P_n d^{t_n} \quad (4)$$

This entire replant cycle will, on average, repeat itself after a period of $t_n = f + a_o + a_1 + \ldots . + a_n$ years, so that the corresponding profits of the various stages of the next replant cycle will each occur t_n years later. In present value terms the total profit of the 2nd replant cycle must be discounted by an additional factor of d^{t_n}, so that we would have a present value of $U_n d^{t_n}$ for the second crop.

Similarly, the profit, in present value terms, for the 3rd replant cycle would have to be discounted by a factor of $(d^{t_n})^2$, giving a present value of $U_n(d^{t_n})^2$. If we sum these profits, discounted to their present values, to infinity, the expression: $Z_n = U_n(1 + d^{t_n} + (d^{t_n})^2 + (d^{t_n})^3 + \ldots\ldots..)$ is obtained.

Examination shows that it is a geometric series to infinity with common ratio = d^{t_n}. Because $0 < d = \frac{1}{1 + R} < 1$, it follows that $0 < d^{t_n} < 1$.

It is shown in Appendix 3 that the sum to infinity of such a series with common ratio between 0 and 1 is a finite number, given by:

$$Z_n = \frac{U_n}{1 - d^{t_n}} \quad (5)$$

Z_n represents the total future profit, discounted to present values, resulting from following a cycle of n ratoons, and starting with a new replant cycle.

Z_n is dependent on the value of n, and the best general ratooning policy on *future* replant cycles is to operate at that average value of n, say N, which give the optimum (maximum) value of Z_n, namely Z_N.

This is simply done by calculating the values of U_n, t_n and

$$Z_n = \frac{U_n}{1 - d^{t_n}} \text{ for each } n = 0, 1, 2, \ldots\ldots,$$

and noting the value n = N which maximizes Z_n.

As an example, Table 1 lists the profits: non-discounted, discounted, and cumulative discounted for the 1st 3 replant cycles of an average field with yields in accordance with Y = 105 — 9r, for the case where the replant cycle length is n = 3.

The cumulative discounted profit Z_3 at time infinitely also is shown.

Figure 3 (*a*) illustrates the profits, non-discounted as well as discounted, as they arise at the times of harvest, and Figure 3 (*b*) shows the cumulative discounted profits.

TABLE 1
Profits per crop for replant cycle length n = 3, starting with plough-out at time = 0

Replant Cycle	Ratoon Stage r	Time at Harvest (years)	Yield (tch)	Profit for Crop: (Rand/Ha) Non-Discounted P_r	Discounted at 15% p.a. to present value: Individual years	Cumulative
1	0	2,08	105	515	385	385
	1	3,66	96	1 028	616	1 001
	2	5,24	87	911	438	1 439
	3	6,82	78	794	306	1 745 = U_3
2	0	8,90	105	515	148	1 893
	1	10,48	96	1 028	238	2 131
	2	12,06	87	911	169	2 300
	3	13,64	78	794	118	2 418 = $U_3(1 + d^{t_3})$
3	0	15,72	105	515	57	2 475
	1	17,30	96	1 028	92	2 567
	2	18,88	87	911	65	2 632
	3	20,46	78	794	45	2 677
∞	Depends on ratoon stage	∞	Depends on ratoon stage	Depends on ratoon stage	0	2 841 = Z_3

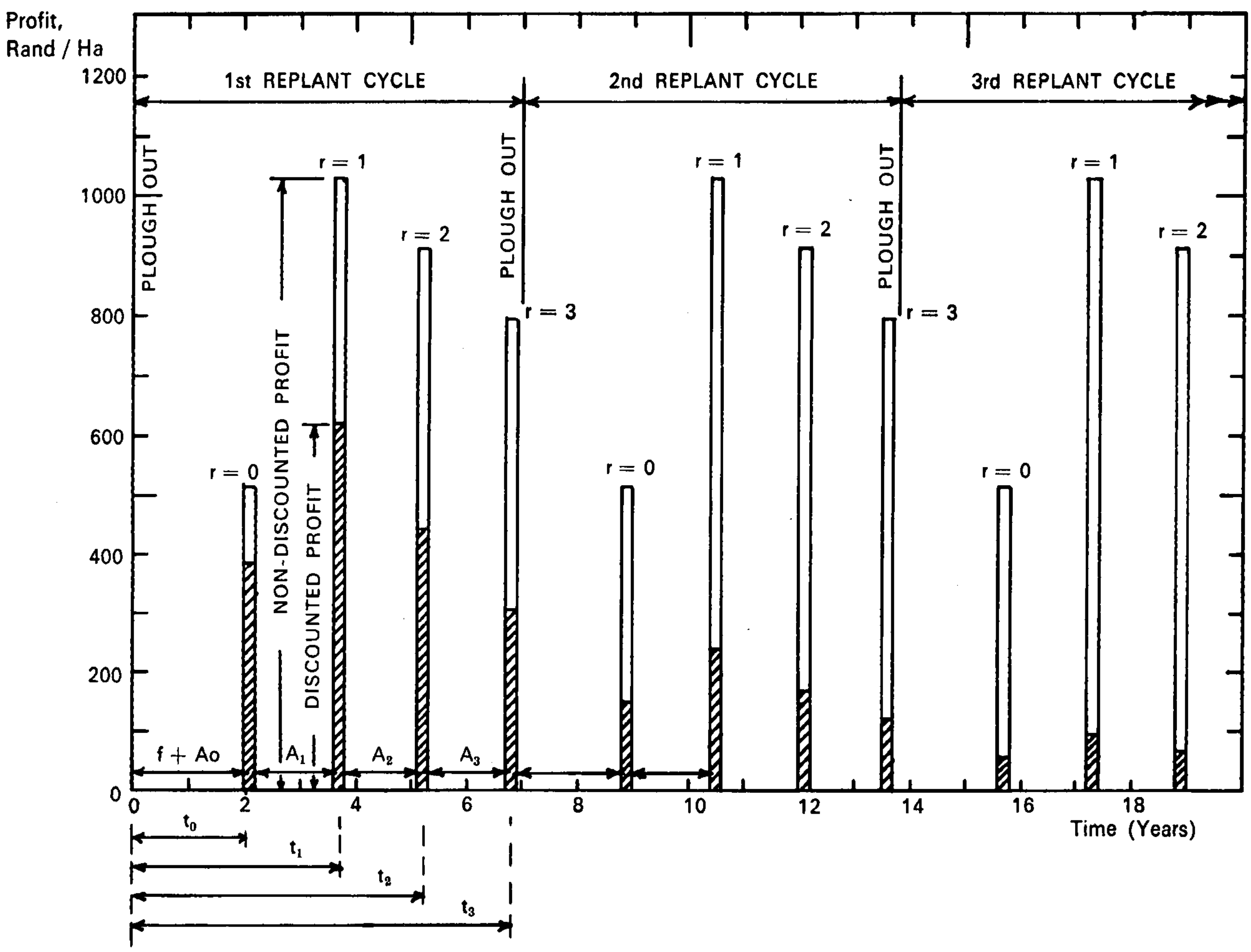

FIGURE 3 (a) Discounted and non-discounted profits per crop, for a replant cycle length n = 3.

TABLE 2

Determination of optimum average number of ratoons N for a field with yield equation Y = 105 − 9r

Costs: Discount rate, R	0,15						
Ratoon cost/Ha, C_H		R220†					
Plant cost/Ha, C_{HP}	R850†						
Harvest cost/ton, C_T	R2						
Value/ton V_T	R15						
Ratoon Stage r or cycle length n	0	1	2	3*	4	5	6
Age (years), A_r	2,08‡	1,58	1,58	1,58	1,58	1,58	1,58
Yield (tch), Y_r	105	96	87	78	69	60	51
Profit (Rand), P_r	515	1 028	911	794	677	560	443
Cum. age (years), t_r	2,08	3,66	5,24	6,82	8,40	9,98	11,56
Discount factor, d^{t_r}	0,75	0,60	0,48	0,39	0,31	0,25	0,20
Discounted profit (Rand/Ha), $P_r d^{t_r}$	385	616	438	306	209	139	88
Discounted profit per cycle (Rand/Ha), U_n	385	1 001	1 439	1 746	1 955	2 094	2 182
Total discounted future profit (Rand/Ha) Z_n	1 527	2 501	2 772	2 841*	2 829	2 784	2 723

† Already Forward discounted to time of harvest ‡ Includes fallow period * Optimum N = 3, $Z_N = Z_3$ = R2 841

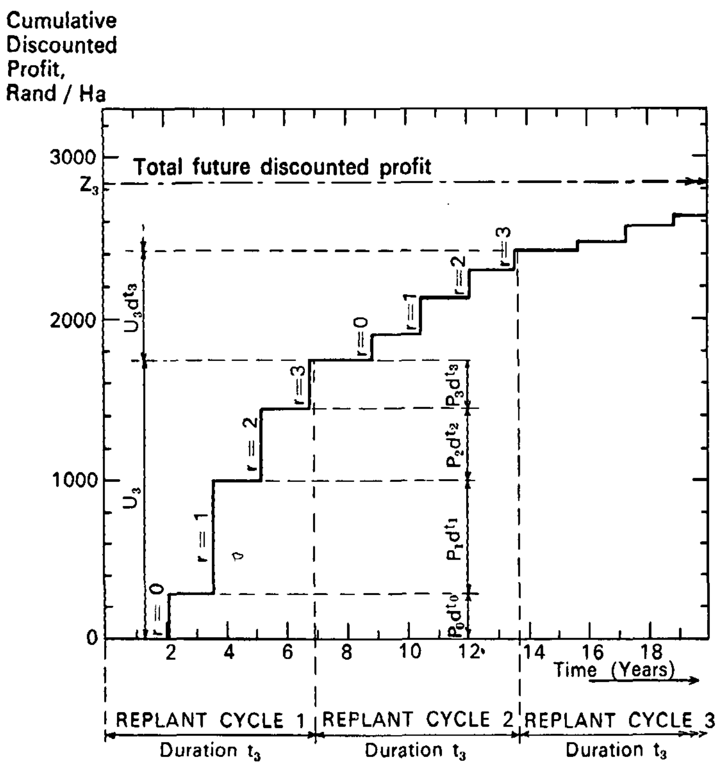

FIGURE 3 (b) Cumulative discounted profits per crop, for a replant cycle length n = 3.

Table 2 shows, step-by-step, how the total discounted profit Z_n is calculated for the various values of replant cycle length n. For the values of input data given, the optimum length of replant cycle is N = 3.

It should be noted that:

(*a*) the optimum is not sharp, because profits which are almost as good can also be obtained at n = 2 and n = 4.

(*b*) the value of N for a specific field will depend on its particular performance parameters and cost rates.

5.2 *Optimum average number of ratoons: In terms of yield equation parameters*

The derivations thus far were general in that the yare valid for any yields Y_r, without the yield equation necessarily having to be linear, i.e. in the form of equation (1). For that matter, they could even have been obtained from a tabulation of representative yield values for the different ratoon stages.

The foregoing results will now be expressed in terms of the parameters A_i and B_i in the linear equation $Y_r = A_i - B_i r$ for a field i

The expressions (2) and (3) for profit

$$P_r = (V_T - C_T) Y_r - C_H, \quad r = 1, 2, \ldots\ldots$$

and $P_o = (V_T - C_T) Y_o - C_{HP}$

for a given field become

$$P_r = (V_T - C_T)(A_i - B_i r) - C_H, \quad r = 1, 2, \ldots\ldots$$

and $P_o = (V_T - C_T) A_i - C_{HP}$

From (4), for average n stages per replant cycle,

$$U_n = P_o d^{t_0} + P_1 d^{t_1} + P_2 d^{t_2} + \ldots\ldots P_n d^{t_n}$$

becomes

$$\begin{aligned} U_n = {} & [(V_T - C_T) A_i - C_{HP})] d^{t_0} \\ & + [(V_T - C_T)(A_i - B_i) - C_H] d^{t_1} \\ & + [(V_T - C_T)(A_i - 2B_i) - C_H] d^{t_2} + \ldots\ldots \\ & + [(V_T - C_T)(A_i - nB_i) - C_H] d^{t_n} \\ = {} & A_i(V_T - C_T)[d^{t_0} + d^{t_1} + \ldots\ldots + d^{t_n}] \\ & - B_i(V_T - C_T)[d^{t_1} + 2d^{t_2} + \\ & \ldots . + nd^{t_n}] - [C_{HP} d^{t_0} + C_H d^{t_1} + \ldots . + C_H d^{t_n}] \end{aligned}$$

From (5),

$$\begin{aligned} Z_n &= \frac{U_n}{1 - d^{t_n}} \\ &= A_i D_n - B_i E_n - F_n \end{aligned} \qquad (6)$$

$$\text{where } D_n = \frac{(V_T - C_T)(d^{t_0} + d^{t_1} + \ldots\ldots + d^{t_n})}{1 - d^{t_n}}$$

$$E_n = \frac{(V_T - C_T)(d^{t_1} - 2d^{t_2} + \ldots\ldots + nd^{t_n})}{1 - d^{t_n}}$$

$$F_n = \frac{C_{HP}\, d^{t_0} + C_H\, d^{t_1} + \ldots\ldots + C_H\, d^{t_n}}{1 - d^{t_n}}$$

The values d, C_H, C_{HP}, V_T and $\{t_r\}$ are fixed for a given field, so the coefficients D_n, E_n and F_n are functions only of n, the replant cycle length.

The optimum recycle length N is found by trying out different values of n in equation (6) and finding which provides the maximum value $Z_N = A_iD_N - B_iE_N - F_N$.

The optimum replant cycle length N and hence the values of coefficients D_N, E_N and F_N are therefore functions of the parameters A_i and B_i for field i.

6. Derivation of plough-out threshold level

6.1 *Decision whether to plough out: General derivation*

Consider a field which has just yielded its nth ratoon stage. If the yields $\{y_r\}$ of its existing replant cycle were to be the same as the estimated average yields $\{Y_r\}$ of future replant cycles, ploughing out would simply be after the Nth ratoon crop.

In general however, the performance of the existing replant cycle would not necessarily be representative of the estimated average yields of future replant cycles. Special conditions such as faulty planting or disease could be the cause that $y_r \neq Y_r$.

If this field is now ploughed out and replanted, the total profit, discounted to present time $t = 0$, will be Z_N, as calculated from equation (5). Here the suffix N and not n is used because it is assumed that the future replant cycles will, on average, all be chosen to be of optimum duration N.

On the other hand, if the field is left to ratoon once more, to yield Y_{n+1} tch at stage $(n + 1)$ a_{n+1} years later, the profit on that crop would be P_{n+1}, plus a total future profit of Z_N, both discounted to time $t = a_{n+1}$. Discounting to the present time $t = 0$, both will be multiplied by a factor $d^{a_{n+1}}$.

Ploughing out and replanting will only be worthwhile if the last mentioned total future profit, discounted to present value, for ratooning once more is less than the value Z_N obtained for ploughing out immediately, i.e. if

$$(P_{n+1} + Z_N)d^{a_{n+1}} < Z_N$$

$$\text{i.e. if } P_{n+1} < Z_N \left(\frac{1 - d^{a_{n+1}}}{d^{a_{n+1}}}\right) \qquad (7)$$

This can be simplified further in terms of the anticipated tons cane per hectare yield y_{n+1} and if we assume that all ratoon crops (excluding the plant crop) are harvested at an average age of a years, so that $a_{n+1} = a$ for $n = 1, 2, \ldots$.

From equations (2) and (7), ploughing out should take place if

$$(V_T - C_T)y_{n+1} - C_H < Z_N\left(\frac{1 - d^a}{d^a}\right)$$

$$\text{i.e. if } y_{n+1} < \frac{Z_N\left(\frac{1 - d^a}{d^a}\right) + C_H}{V_T - C_T} \qquad (8)$$

The right hand side of the equation represents the *plough-out threshold level*, and with the exception of Z_N, all the values are fixed constants. Z_N is dependent on the expected yields Y_0, Y_1, ...from subsequent replant cycles, and will be constant for a given field. In that event the right hand side of equation (8) is a constant, say Q = Plough-out threshold level.

Condition for plough-out then becomes:

$$y_{n+1} < Q$$

In our example,

$$Q = \frac{2841\left(\frac{1 - (0{,}87)^{1.58}}{(0{,}87)^{1.58}}\right) + 220}{15 - 2}$$

$$= 70{,}9 \text{ TCH}$$

Therefore, if the next ratoon crop is expected to yield below this figure, the field should rather be ploughed out and replanted.

6.2 *Remarks*

(*a*) If a field is planned to be ratooned once more, then each tch that its yield will be below its plough-out threshold, signifies a present-value loss of:

Marginal profit/ton of cane × Discount factor from harvest time to present time

$= (V_T - C_T)d^a$ Rand/tch

In our example, the loss would be:

$(15 - 2) \times (0{,}870)^{1.58}$

= R10,40 per hectare per tch yield below plough-out threshold level.

The same value applies for every tch that a ploughed-out field would have yielded above its plough-out threshold, had it been allowed to ratoon once more.

(*b*) When discounting is not used, the optimum replant cycle length N remains at 3, but the plough-out threshold value for the example is 77,1 tch; considerably different from when a discount rate of 15% was used. This goes to show that discounting is not merely done for the sake of academic rigour.

6.3 *Decision whether to plough out: In terms of yield equation parameters*

Substituting equation (6) into the condition (8) for plough-out of field i:

$$y_{n+1} < \frac{Z_N(1 - d^a)}{d^a(V_T - C_T)} + \frac{C_H}{V_T - C_T}$$

$$\text{i.e. } y_{n+1} < A_i\left(\frac{D_N(1 - d^a)}{d^a(V_T - C_T)}\right) - B_i\left(\frac{E_N(1 - d^a)}{d^a(V_T - C_T)}\right) - \left(F_N\frac{(1 - d^a)}{d^a(V_T - C_T)} - \frac{C_H}{V_T - C_T}\right)$$

If we write

$$H = \frac{D_N(1 - d^a)}{d^a(V_T - C_T)}$$

$$K = \frac{E_N(1 - d^a)}{d^a(V_T - C_T)}$$

$$L = \frac{F_N(1 - d^a)}{d^a(V_T - C_T)} - \frac{C_H}{V_T - C_T}$$

the expression for plough-out becomes:

$$y_{n+1} < A_iH - B_iK - L = Q \qquad (9)$$

For fixed values of discount rate d, costs, and average age a, the coefficients H, K and L will depend on D_N, E_N and F_N, which in turn depend on A_i and B_i. H, K and L can therefore be considered as functions of A_i and B_i, and so can the plough-out threshold level Q, by equation (9).

Because optimum number of ratoons N also is a function of the parameters A_i and B_i, it is instructive to graphically show how the combinations of A and B affect the values of N and Q.

The equation of the boundary line between the regions of $N = n$ and $N = n + 1$ is obtained by setting:

$Z_n = Z_{n+1}$

i.e. $A D_n - B E_n - F_n = A D_{n+1} - B E_{n+1} - F_{n+1}$

$$\text{or } B = \left(\frac{D_n - D_{n+1}}{E_n - E_{n+1}}\right) A - \frac{F_n - F_{n+1}}{E_n - E_{n+1}} \qquad (10)$$

This is the equation of a straight line with A and B as variables, and these boundary lines are shown as dotted in Fig. 4.

The equation for a line expressing the relationship between all values of A and B which yield the same plough-out threshold level Q is found by using equation (9):

$A H_n - B K_n - L_n = Q,$

$$\text{from which } B = \frac{A H_n}{K_n} - \left(\frac{L_n}{K_n} + \frac{Q}{K_n}\right)$$

This is a straight line equation in A and B over the range for which n = optimal N, and these are shown in Fig. 4 as solid lines for various values of constant Q.

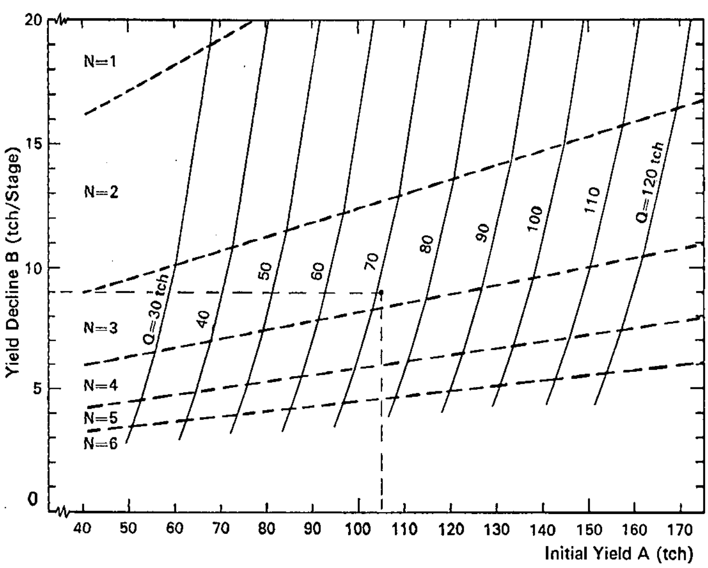

FIGURE 4 Optimum number of ratoons N and plough-out threshold level Q as functions of yield equation parameters A and B.

An interesting characteristic is that the optimum number of ratoon stages N is more sensitive to the values of ratoon decline rate B than to initial yield A, whereas the plough-out threshold level Q depends more on A. This last-mentioned aspect implies that, for the purpose of knowing what to do next with a field, accuracy in the value of its parameter A is more important than of B.

Applying the parameters A = 105 and B = 9 from our example to the graph, the values of N = 3 and Q = 71 approximately are read off, which is in agreement with the previous calculations.

7. Use of historical yield data from fields

7.1 *Estimation of the yield parameters for a field*

For a given field i it is required to know the most likely values of its parameters A_i and B_i in *future* crops.

We should not confine ourselves to the results of the present replant cycle, but use as much previous data as possible, all of it converted to standard yields.

The obvious way to calculate the parameters A_i and B_i is by applying linear regression analysis to the historical standard yield data of each individual field. A problem however arises when, notwithstanding the attempts to convert the yields to conditions of standard climate and age, the scatter of the points of some fields is so high that these parameters cannot be determined with any reasonable degree of reliability. In section 6.3 it was already graphically shown that accuracy in the parameter A is more important than in B.

To investigate this aspect further, each of the following 2 hypotheses was statically tested:

Hypothesis 1

All fields have the same intercept A for their yield equations, irrespective of their individual slops B_i.

Hypothesis 2

All lines have the same slope for B for their yield equations, irrespective of their individual intercepts A_i.

An analysis of variance on the fitted regression lines was done for each of the hypotheses, as per Rao[3], and the results were as follows:

	Calculated F-test statistic	Degrees of Freedom		F-statistic at 0,1 % level of significance
		V_1	V_2	
Hypothesis 1	3,36	514	1417	1,21
Hypothesis 2	1,37	514	1417	1,21

Both these hypotheses could be rejected at the 0,1 % level, indicating that, in principle, each field has its own yield equation, but Hypothesis 1 can be rejected with a far higher degree of confidence.

This means that when the line fit for a field is poor, we would be making less of an error in assuming equality of the B-parameters than of the A-parameters.

In estimating the parameters, both the following methods should therefore be used:

(i) Fitting a straight line by linear regression, which will provide values for A_i and B_i

(ii) Fitting a straight line by linear regression, but constraining its slope to the standard value — B.

The respective 95 % confidence limits for both estimated values of A_i should be determined, and the parameter values corresponding to the method with the better confidence limits should be used for that field.

7.2 *Estimation of next crop yield in the event of ratooning once more*

The potential yield y_{n+1} of the next ratoon crop in that replant cycle should be estimated only from historical data pertaining to the present replant cycle, because the existing root system might have some peculiarities which are not typical of that field's usual performance. It is only necessary to estimate one ratoon stage ahead.

In estimating the standard yield for stage n + 1, 3 methods are available:

(i) Fitting a straight line by linear regression, and extrapolating

(ii) Fitting a straight line by linear regression, whilst constraining its slope to the average value B, and extrapolating

(iii) Extrapolating by free-hand.

In each of methods (i) and (ii) the 95% confidence levels of the extrapolated yield y_{n+1} should be determined, and the y_{n+1} with the better confidence limits chosen for comparing with the plough-out threshold level Q.

Option (iii) has merit in that the estate manager has in his head non-quantifiable knowledge of the field, which he would take into account when making the extrapolation. If he is sufficiently confident he can ignore the computed estimates and use his own estimate of y_{n+1}.

8. Use of the computer

For the practical application of this system the calculations involved would be very tedious to do by hand, and an ICL 1902A computer was therefore programmed to process the data. A sample of the computer print-out is given.

8.1 *For all fields*

A listing of data common to all fields gives the relevant seasonal climatic yield correction factors for each estate and past season; the basic costs for establishing a ratoon crop, establishing a plant crop, harvesting and transport; the revenue per ton of cane; the desired rate of return on capital; average fallow period; average harvest ages and the standard rate of decline. This is shown in Fig. 5 (*a*).

8.2 *For each field*

All the historical performance data on that field are listed and these yields are then corrected for seasonal climatic conditions and age to provide the standard tch values — refer to Fig. 5 (*b*).

The plough-out threshold level is determined by both methods, as described in Section 7.1, from all the historical data available. The program will recommend that value with the superior confidence level — in this example, the value obtained by constraining the slope.

The computer program also estimates the potential yield of the next ratoon crop of the existing replant cycle by both methods, using only the historical data for the existing replant cycle, as described in Section 7.2.

```
#RTN4        HULETTS SUGAR LTD.  -  DETERMINATION OF FIELD PLOUGH-OUT THRESHOLD - DARNALL ESTATES.
                                                                                        29/03/76
   INPUT DATA USED FOR THE ANALYSIS:

     COSTS,   RAND/HECTARE  : CH  - FOR ESTABLISHING A RATOON CROP :  220,00

                              CHP - FOR ESTABLISHING A PLANT  CROP :  850,00

     COSTS,   RAND/TON CANE : CT  - FOR HARVESTING, TRANSPORT ETC  :    2,00

     REVENUE, RAND/TON CANE : VT  - INCLUDES EXTRA PROFIT TO MILL  :   15,00

     DESIRED RATE OF RETURN ON CAPITAL :   15,00 % PER ANNUM

     CLIMATIC YIELD CORRECTION FACTORS :
```

	ESTATE										
SEASON	1211	1212	1213	1214	1215	1216	1221	1222	1225	1231	1232
	NONOTI	OCN VW	PROSPC	SINKWZ	SPRNGF	TUGELA	COLERN	HOLWD	SPROWS	CLIFTN	Z.S.M
65/66	0,842	1.438	1.469	1.478	1.120	1.125	1,981	1,114	1,278	3.259	0.000
66/67	0,956	0,932	1,000	1.083	0.944	1,027	1.188	0,978	1,208	1.617	0,836
67/68	0,951	0,922	0,852	1,112	0,981	1,061	1,079	0,924	1,103	0.967	0.945
68/69	0,820	0,838	0.901	0,896	0.909	0,884	1,037	1.044	1,110	0.978	0.921
69/70	0,967	0,942	0,931	0,987	0,891	0,935	1,082	0,933	0,956	1.044	0.933
70/71	1,408	1,042	1,377	1,222	1,101	1.170	1,003	1.233	1,016	1.107	1.158
71/72	1,020	1,047	0,951	1,012	1,012	0,936	0,924	1,004	1,060	0.929	0,953
72/73	1,078	1,004	0,962	0,901	0,998	1,014	0,933	0,960	0,899	0.903	1.032
73/74	1,030	0,975	0,952	1,001	1,008	0,974	0,929	0,969	0,939	0.875	1,012
74/75	1,006	1,162	1,050	0,980	1,125	1,013	0,948	0,958	0,879	1,038	1,045

```
     STANDARD FALLOW PERIOD              :   5,00 MONTHS

     STANDARD HARVEST AGE : PLANT CROP  :   20,00 MONTHS
                           RATOON CROP  :   19,00 MONTHS

     STANDARD RATE OF YIELD DECLINE      :   9,00 TCH DROP PER RATOON STAGE
```

FIGURE 5 (a) Computer print-out: Input data common to all fields.

Lastly, for each field the computer provides a graphical representation of past crop standard yields and plots the fitted line upon which the plough-out threshold was calculated. It also shows the plough-out threshold level and the estimated next-ratoon crop yield — refer to Fig. 5 (*c*).

This particular example shows why it is desirable to use only data from the existing replant cycle for extrapolating to the next ratoon crop yield. The existing root system appears to have something wrong with it, because both previous crops (plant and first ratoon) had yields well below the corresponding crops from the previous replant cycle. Plough-out is indicated, even though that replant cycle had thus far only yielded one ratoon crop, in contrast to the average optimum of N = 3.

9. Conclusion

This system provides field management with a rational method for deciding whether or not to plough-out a field, without involving them in any calculating work. Obviously there will also be other considerations, such as availability of labour, which determine the nature of the planned field operations, but at least this information will point out to field management the more urgent cases, so that they can allocate scarce resources to such fields and leave the borderline cases over for one more ratoon crop.

10. Acknowledgements

My thanks go to Peter Dovey for supplying the necessary data and to Trevor Ireland for his work in analysing the data and designing the computerised processing system.

REFERENCES

1. Boyce, D. S. (1966). To determine the most profitable number of sugar cane crops between replanting. Sugar y Azucar **63**: 32-33.
2. Du Toit, J. L. (1960). Some items of economic importance in sugar cane production. SASTA Proc **34**: 183-188.
3. Rao, C. R. (1965). Linear statistical inference and its applications. Wiley, New York. p 281-284.

#RTN4 HULETTS SUGAR LTD. * DETERMINATION OF FIELD PLOUGH-OUT THRESHOLD - DARNALL ESTATES.

29/03/76

ESTATE 1212 OCEAN VIEW FIELD 110 TRIANGLE

STAGE	T.C.H.	AGE	CUT DATE	SEASON CORR FACTOR	STD AGE	AGE CORR FACTOR	STANDARD T.C.H.
0	123.29	28	9/66	0.932	20.00	0.845	97.11
1	106.73	18	3/68	0.922	19.00	1.027	101.10
2	89.67	20	11/69	0.942	19.00	0.975	82.33
3	77.53	20	7/71	1.047	19.00	0.975	79.12
0	72.71	18	10/73	0.975	20.00	1.054	74.73
1	45.84	16	2/75	1.162	19.00	1.090	58.05

FITTING OF REGRESSION LINES, WITH 95% TOLERANCES ON PARAMETERS :

DETERMINATION OF PLOUGHOUT THRESHOLD (USING ALL HISTORICAL DATA) :

METHOD OF FITTING :	1. OWN SLOPE	2. CONSTRAINED SLOPE
INITIAL YIELD (INTERCEPT) A :	84.35 ± 29.11	92.57 ± 18.43
RATOON DECLINE (SLOPE) B :	1.95 ± 18.41	9.00 (CONSTRAINED)
PLOUGH-OUT THRESHOLD :	64.23	59.86

DETERMINATION OF NEXT RATOON CROP YIELD (USING ONLY PRESENT REPLANT CYCLE DATA) :

METHOD OF FITTING :	1. OWN SLOPE	2. CONSTRAINED SLOPE
INITIAL YIELD (INTERCEPT) A :	0.00 ± 0.00	70.89 ± 48.82
RATOON DECLINE (SLOPE) B :	0.00 ± 0.00	9.00 (CONSTRAINED)
NEXT RATOON CROP YIELD :	0.00 ± 0.00	52.89 ± 69.04

USE CONSTRAINED SLOPE TO CALCULATE PLOUGH-OUT THRESHOLD : 59.86
USE CONSTRAINED SLOPE TO ESTIMATE NEXT RATOON CROP YIELD : 52.89

RECOMMENDATION : PLOUGH OUT.

FIGURE 5 (b) Computer print-out: Input data and results for a specific field.

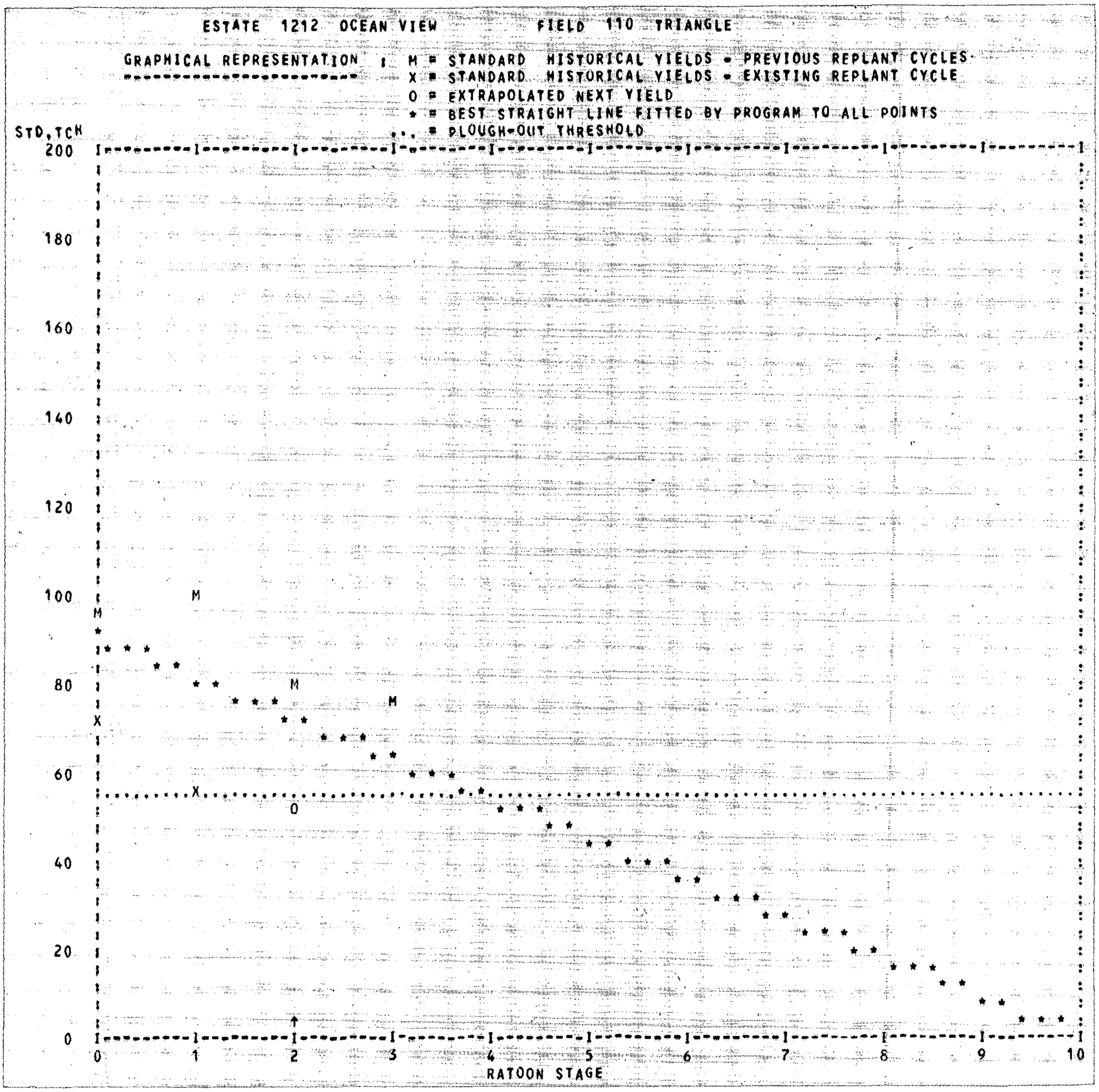

FIGURE 5 (c) Computer print-out: Graph plot of performance for a specific field.

Appendix 1

Correcting for age in crop yields

For each season j of a specific state e, define season correction factor

$$K_{ej} = \frac{\text{Average tchm for all crops of estate e, all seasons}}{\text{Average tchm for all crops of estate e in season j}}$$

and age correction factor for crop j of field i

$$(K_A)_{ij} = \frac{\text{Standard age}}{\text{Actual age}}$$

where Standard age = 20 months for plant crops
= 19 months for ratoon crops

Define standard tch as

$$Y_{ij} = (\text{Actual tch})_{ij} \times (K_{ej})_i \times (K_A)_{ij}^W$$

where w lies between zero and 1

Choosing a value of w, say w = 0,3, a regression equation of the form:

$$Y_{ij} = A_i - B_i r_{ij}$$

was fitted to the standard tch's of each field, where r is the ratoon stage. The parameters A_i and B_i will be specific for each field i

The sum of squared residuals

$$SSR_W = \sum_i \sum_j \left(Y_{ij} - (A_i - B_i r_{ij})\right)^2$$

was calculated for that value of w

The entire procedure was repeated for the values of w = 0, 0,3, 0,5, 0,7 and 1,0, resulting in the tabulation:

Exponent w	0	0,3	0,5	0,7	1,0
$SSR_W \times 10^3$ . . .	517,1	428,8	402,3	403,6	464,5

The lowest sum of squared residuals was obtained for an exponent of approximately w = 0,55, so that it was decided to let the correction for seasonal climate and age take the form of:

$$Y_{ij} = (\text{Actual tch})_{ij} \times (K_{ej})_i \times \sqrt{(K_A)_{ij}}$$

The reason for fitting a separate regression equation to each field and summing up the respective squared residuals instead of fitting a single equation to all the data, is the large difference in the individual fields' performances, and to eliminate the effect of pre-selectivity — See Section 3.2.

Appendix 2

The principle of discounting to present value

Suppose you have to choose between either receiving an amount of money P at the present time (taken to be year 0) or alternatively receiving an amount of money S at a time n years later. The question arises:

> What should the relationship of these respective amounts be to make it immaterial which alternative you chose, i.e. so that you would be equally well off for either of the 2 possibilities?

Supposing you chose the first possibility, of receiving amount P at a time 0, and that you did not spend it, but invested it at a compound interest rate of R per annum, where R is expressed as a fraction and not a percentage. In n years' time this amount would have grown to $P(1 + R)^n$. Had you chosen the second alternative, you would, also in year n, be in possession of an amount S. The two alternative choices would therefore be of equal value to you if these two amounts at time n were equal, i.e.

if $S = P(1 + R)^n$

Re-arranging the terms we have:

$$P = \frac{S}{(1 + R)^n}$$

Therefore, the amount S received in n years' time has a *present value* of $\frac{S}{(1 + R)^n}$ and is said to have been *discounted* by an amount $\frac{S}{(1 + R)^n}$

The factor $\frac{1}{1 + R} = d$ is called the *discount factor*, and to calculate the present value of any amount receivable or payable in year n, that amount is multiplied by the discount factor raised to the power n, i.e. multiplied by d^n.

Appendix 3

Determination of the sum of a Geometric series

Consider the series

$$S_n = 1 + u + u^2 + \ldots\ldots + u^{n-1} + u^n$$

where S_n signifies the value of the sum of terms up to u^n. u is called the *common ratio.*

Multiplying the equation by u, we obtain:

$$uS_n = u + u^2 + u^3 + \ldots\ldots + u^n + u^{n+1}$$

Subtracting the second from the first equation, we obtain:

$$(1 - u)S_n = 1 - u^{n+1}$$

from which follows:

$$S_n = \frac{1 - u^{n+1}}{1 - u}$$

If $-1 < u < 1$, we know that

$u^{n+1} \rightarrow 0$ as $n \rightarrow \infty$

Representing the value of the sum to infinite terms by S_∞, we have that:

$$S_\infty = \frac{1}{1 - u}$$