# THE LOGISTIC CURVE OF PLANT GROWTH AND ITS APPLICATION TO SUGARCANE

By A. McMARTIN


## Abstract

Quantitative data on the rate of development of a sugarcane crop show that the pattern follows that of the logistic curve of plant growth. In this, a period of fast growth changes to one of slow growth at a defined point known as the point of inflection. Data from measurements made at the Sugar Corporation of Malawi have shown that a formula equating the rate of growth with that of an autocatalytic chemical reaction can be applied, by means of which the final growth is predicted by calculation from the inflection point. The position and magnitude of the latter is of significant importance in the curve, and there is evidence that the ultimate effect of factors influencing growth is governed by the time of their occurrence in relation to this point.


## Introduction

Research by various investigators into the rate of plant growth has focused attention on the fact that a definite pattern is always recognised and is similar in form whether the study be of the growth of a single organ, or of the plant as a whole or of the rise in numbers of an increasing plant population. This is simply a growth pattern which conforms to the general mathematical form of growth of all living organisms and populations. (Among literature on this subject reference may be made to D'Arcy W. Thompson[11] for a discussion on growth rate studies applied to biological phenomena generally and to Sanderson[10] for the relevance of plant growth rate data to crop forecasting.)

Expressed graphically by plotting the item of development being studied, such as length, total mass or dry matter, over time, two types of curve may be produced. Firstly, the increment curve, which is derived from the difference between measurements made on successive occasions and shows simply a steady increase in periodic gains up to a maximum point followed by decrease to the cessation of growth, and secondly, the summation curve which is the total measurement at each period and is characterised by well-defined changes in slope. At the start the curve is slow and commonly referred to as the "lag" phase, leading into a phase of rapidly increasing growth in which each succeeding increase is greater than the previous one — a phase of logarithmic increase. This leads into a phase of decreasing increase, or "negative acceleration", ending with a final period of slow growth. At a point in the rising curve the change from fast growth to slower growth occurs, known as the point of inflection. In a simple curve this point occurs at or near the centre of the curve, and the whole curve is symmetrical in nature, with the growth made in the second half being equal, or approximately so, to that in the first half. The increment curve is bell-shaped in pattern, and the summation curve S-shaped or sigmoid. This sigmoid type of curve was at an early date named the logistic curve of plant growth, and the law according to which plant increase occurs has been referred to as the normal law of plant growth.

Deviations, however, occur in the symmetry of the curve, particularly with the position of the point of inflection. The description given refers to the simplest form of curve, but this simplicity does not always obtain. Thus, the point of inflection may occur to the left or right of the half-way point of total growth, resulting in curves which are asymmetrical in form. Moreover the regular phasic development may not be exhibited. A common example of the latter is found in the field of microbiology. With brewer's yeast, it is known that when introduced into malt it shows a lag phase of one or two hours, followed by a logarithmic increase, then a period of retardation and final cessation of growth. If, however, seeding is by yeast already actively growing, the lag phase disappears, while if seeding is of older growth there is no logarithmic phase and retardation sets in immediately. In such cases the point of inflection occupies different positions in the curve. (This will be referred to later with sugarcane germination.)

## Growth curves and sugarcane

Sugarcane growth measurements and curves have been dealt with by van Dillewijn,[2] Koenig,[3] Willcox,[12] and Sanderson,[10] and the writer.[5-9]

*Germination and early growth* Investigations into germination and early growth have showed that the rate of emergence of buds when planted followed the logistic curve pattern and moreover that buds from the younger part of the stem could omit the initial slow growth phase, with an earlier inflection point in the curve in addition to ending with a higher percentage of buds germinated (Fig. 1).

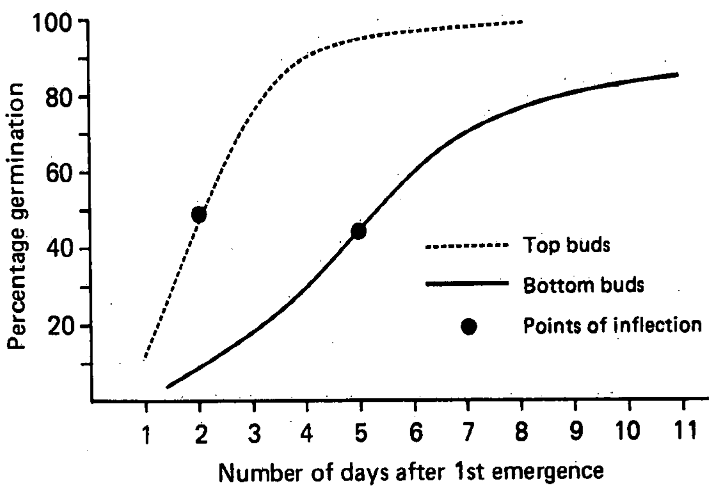


**FIGURE 1** Germination of buds from top and bottom parts of cane.

Trials carried out with various fungicides as a preplanting treatment for cuttings showed that it was possible to use a treatment which as a protectant against disease was extremely effective, judged by a final germination count, but that a treatment which resulted in a lower count, but encouraged a faster emergence rate, was in the end productive of a greater density of young shoots. Quicker germination was followed by faster tiller production. This is shown in Fig. 2 where two organo-mercurial fungicides were compared in a field plot trial. It was found in one trial in which this early difference was very pronounced that a significant increase in final yield was obtained. (The fungicide Aretan was recommended on its performance as a promoter of rapid and satisfactory germination, despite the fact that other materials were available which showed a higher final germination count, but for which development rate was slow.)

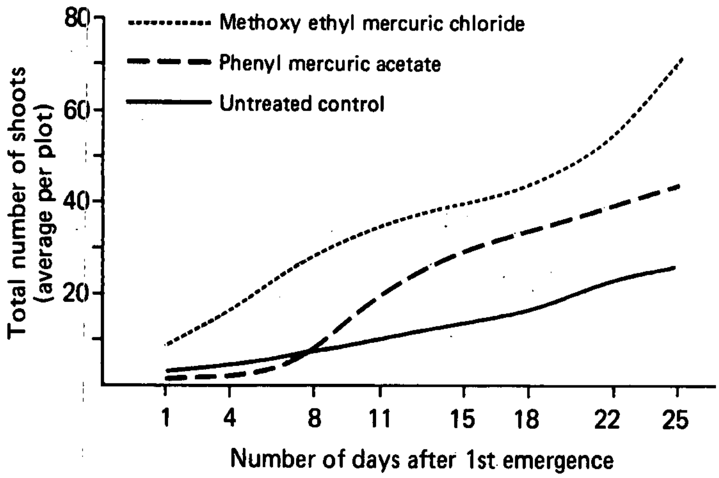


**FIGURE 2** Effect of preplanting fungicidal sett treatment on shoot development.

The advisability of having not only a satisfactory final germination count but also a quick rate of emergence was early recognised in the seed trade, where two figures have been used to denote quality of seed samples. The final count of germination has been denoted germination capacity, while the term germination energy has been used for the germination count after a given number of days, which varies with the crop.

*Growth rate of the cane stem* It has become an increasing practice on estates to make periodic measurements of the rate of cane elongation as a means of obtaining some concrete data on the progress of crop growth. This is particularly so on irrigated estates and is usual as a means of checking the water application, for example detecting period of moisture stress and regulating the irrigation schedule accordingly. Details of the actual procedure are not given here, nor are the pros and cons of the reliability of any technique discussed, but the general practice is simply to measure selected canes from a fixed point at the bottom to the top dewlap. Interesting examples of growth rate measurements plotted graphically are shown by van Dillewijn.[2] These were derived from data obtained from the collections of sugarcane varieties (arranged according to species) grown at Canal Point, Florida, and at Summit, Panama Canal. It is seen that in one locality a symmetrical curve was produced, while the other was asymmetrical, a difference attributed by Brandes to variation in day-length.

Growth measurements have been the practice at the estate of the Sugar Corporation of Malawi at Nchalo. This estate has been developed in an area of stratified alluvium in the valley of the lower Shire river. Here the rainfall is insufficient by itself to produce a satisfactory crop, thus a complete irrigation schedule is followed. Temperatures are high and the crop is an annual one. A total area of 9 067 hectares has been developed from 530 hectares harvested in 1966 under an overhead irrigation system. The area is flat and laid out on a uniform design with now over 320 fields all equal in size. Each field has had two cane measuring locations, where measurements have been made twice weekly, and the measurements have been totalled and presented monthly. Thus over the whole area an exceedingly large amount of data has become available. (The total number of measurements has been reduced recently.) At the end of the 1977 season the writer undertook the plotting of the growth measurements of crops harvested. The harvest season is from April to November, and the season of active growth starts in July and ends in June. Figs. 3 and 4 show the growth rate curves since the start of the estate, when at a very early stage growth measurement stations were inaugurated. These curves of the average monthly growth from 1966 to 1978, are of interest, as they cover all conditions from cane planted after bush clearing to cane replanted after a crop was ploughed

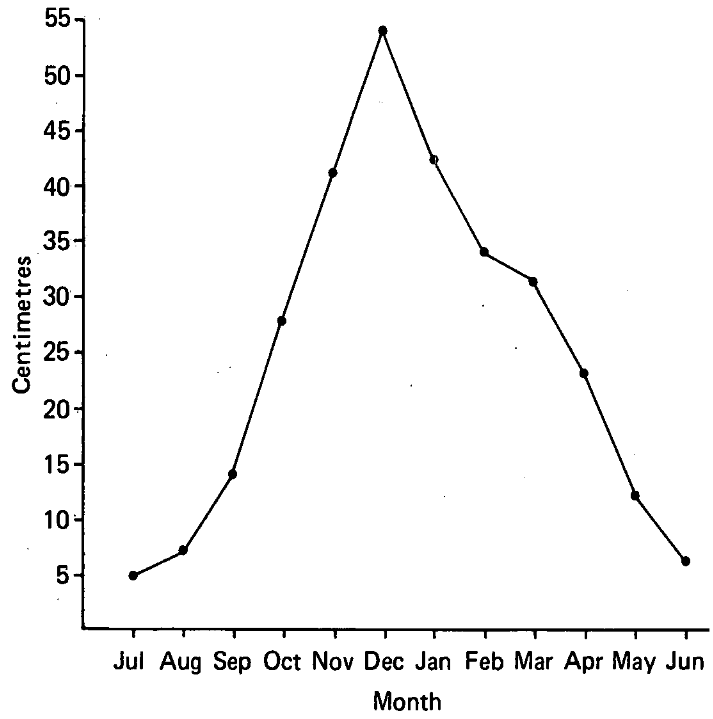


**FIGURE 3** Sugar Corporation of Malawi, Nchalo Estate. Increment growth curve of sugarcane stems. Monthly average for 12 years, 1966–1978.

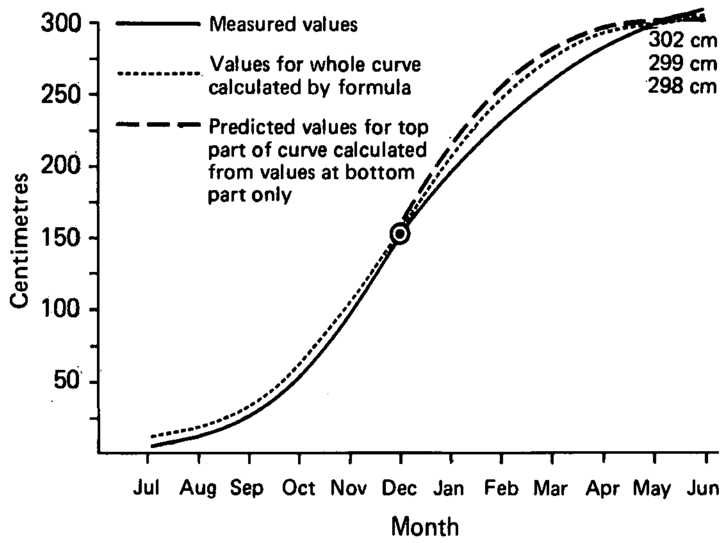


**FIGURE 4** Sugar Corporation of Malawi, Nchalo Estate. Summation growth curve of sugarcane stems. Monthly average for 12 years, 1966–1978.

out, all times of planting, crops from plant cane to 12 th ratoon, periods of water shortage and others of excess and a variation in soil types. Different varieties are involved; these consisted in the 1978 crop of 68 per cent of NCo 310, 25,6 per cent of NCo 376 and the remainder of small plots of others under field trial.

The average increment growth curve for the whole period is shown in Fig. 3. Cane elongation started with a monthly growth of 4,9 cm for July, reaching a peak of 54,3 cm in December and dropping to 6,3 cm in June. A regular and smooth upward slope of the curve is shown, but in the downward curve a break in its even course is found in the February to March slope. The summation curve is presented in Fig. 4. Here a typical sigmoid logistic curve is shown, with a slow start passing into a period of fast increase which leads into a period of slower increase and a final retarded growth phase.

The point of inflection is at the end of December. The final height is 301,8 cm and the point of inflection is at the height of 150 cm. It is thus practically central, both with regard to the time of occurrence and the growth achieved.

### Mathematical studies of growth curves

Since an early date in the studies of growth curves of biological phenomena, attempts have been made to find a formula which would be applicable to the data. Such ones are the compound interest law of plant growth enunciated by Blackman, the autocatalytic chemical reaction rate conception of Robertson, and others developed by Mitscherlich and Baule and their interpretations by Spillman and Willcox. The use of growth rate formulae for sugarcane specifically has been referred to by Willcox,[12] van Dillewijn,[2] and in Mauritius Koenig[3] fitted a logistic curve and formula to cane height measurement, taking into consideration variations due to soil moisture and air temperature. The Robertson formula was referred to by the writer earlier (McMartin[6]) and lately was used on the figures obtained from Sucoma at Nchalo. This arose from seeing the results of using this formula by Reed and Holland on the growth of sunflower, quoted by D'Arcy Thompson.[11] This formula has been referred to as being applicable where a simple symmetrical logistic curve is involved, and is founded on the theory that the rate of growth of this pattern is the same as that of a physico-chemical reaction of the autocatalytic type.

The formula is:

$$\log \frac{x}{a - x} = K(t - t')$$

where x = height at any given period
a = maximum height attained
t = period of time at which height x was reached
t' = period at which one half of the maximum height was reached
K = a factor, the constant of proportionality, to be calculated.

As a first step in testing the suitability of this formula for the Nchalo figures the factor K was determined by using the actual monthly figures of cane height measurements.

The average K for the year was 0,311. Using now this figure, the height of growth for each month was calculated by the formula and plotted alongside the graph of observed values. This is shown on Fig. 4 (dotted line). It was apparent that this equation gave a good fit to the actual measured height. The next step was to take the value of K for the first half of the growth curve only and apply it to the second half of the curve. This is shown as the broken line along the curve in Fig. 4. A comparison of the figures for final height and the rates of growth are shown in Table 1.

**TABLE 1**

**Nchalo: derivation of average maximum cane heights by different means 1966–1978**

| Origin of figure | Maximum height: cm |
|---|---|
| Actual measurement . . . . . . . . . . | 302 |
| Calculated by formula for the whole growth period | 298 |
| Calculated by formula for first half of growth period | 299 |

By using the observed data from the start of the growing season until the half-way point, the ultimate height achieved was forecast at this time with a reasonable degree of accuracy. This was done again with the 1976–77 and 1977–78 crops, when from the mid-point measurement reached, the final height was forecast. The results are shown in Table 2.

**TABLE 2**

**Nchalo: derivation of maximum heights for 1977 and 1978 by different means**

| Origin of figure | Year | Maximum height: cm |
|---|---|---|
| Actual measurement . . . . . . | 1976–77 | 305 |
| | 1977–78 | 303 |
| Forecast by formula from the first half of growth period . . . . . . . . | 1976–77 | 302 |
| | 1977–78 | 300 |

It would appear then that the application of this formula to early growth data provides a means of forecasting the future behaviour of the crop. It should be pointed out, however, that at Nchalo, with its annual crop and control of moisture an ideal set of conditions obtain for producing a simple growth curve which is suitable for mathematical interpretation. Measurements carried out under conditions for slower growth due to temperature and natural rainfall, involving insufficiency of water, resulting in crops with growth during more than one season produce curves of a bi-modal pattern and greater variability which are more difficult to study mathematically.

One matter of interest may be mentioned here. It was pointed out in the increment growth curve in Fig. 3, that an irregularity in the downward slope is noticeable in February and March. In fact in some years this is extremely pronounced, to the extent that an actual dip occurs at this period, with the March reading being higher than the February one. The effect of this is noticeable in the summation curve, where over a period the actual measurements may not fit closely with the expected ones derived by calculation, although at the end of the curve they meet. This is a matter being investigated but the opinion is being formed that this phenomenon is associated with the period where lodging (which can be severe at Sucoma) commences, and also is at the time of change from vegetative growth to flowering.

### Cane length in relation to yield

While cane length is a component of final yield and is a relatively easy method of measuring the progress of a crop, it must of necessity be taken into account that other factors such as density of canes per unit of area, and mass of individual canes all play a contributory part in determining the ultimate mass per unit of area. These in turn are related to such factors as age and season, and naturally different varieties have a different pattern of behaviour. This is clearly shown in the paper by Lonsdale and Gosnell.[4] Nevertheless it was considered to be of interest to determine whether or not the data obtained from cane measurements could be used as a guide to the yield obtained. For this purpose, the simple procedure was tried of expressing the unit of length grown as an index of final yield.

To obtain a figure to apply to the calculated length of cane grown in 1977, 302 cm, the average tch per cm of cane of the two previous years was taken.

This gave an average for the two years of 0,310 tch per cm of cane grown. Applying this now to the figure predicted in January 1977 for the ultimate length of cane in the forthcoming harvest, the following was derived — 302 cm of cane at 0,310 tch per cm of cane is 93,53 tch. This would be for a 12-month crop, representing 7,79 tch month. The actual yield was 88,2 tch for 11,79 months or 7,48 tch month. A close agreement was thus reached in this one case by this empirical method. The matter was, however, investigated further. The harvest season at Sucoma covers the period from April to November, and an analysis was made of the yield per unit of length on a monthly basis, as shown in Table 3.

TABLE 3
**Nchalo: Growth rate and yield on a monthly basis: 1977 crop**

| Month of harvest | tch | tch per cm of cane |
|---|---|---|
| April | 105,9 | 0,331 |
| May | 98,1 | 0,333 |
| June | 91,6 | 0,331 |
| July | 90,5 | 0,309 |
| August | 86,9 | 0,303 |
| September | 85,8 | 0,303 |
| October | 82,7 | 0,284 |
| November | 80,6 | 0,266 |

Thus a yield decline obtained as the harvest progressed, as did the yield per unit of length of cane grown. Factors suggested as possibly contributing to this condition are variation in cane population per hectare, loss of mass in individual canes either due to lodging, flowering, or loss due to a reduction in thickness. That any of these occur, however, has not been established, but the matter is the subject of further investigation. A further study was made of the yield related to length on an estate area basis. Sucoma in 1977 was divided into 4 managerial areas (this is now 5), and the yields from these areas were treated similarly.

The cane growth was, as may be expected, not uniform throughout the whole estate. Three out of the four areas had an average growth rate of 25,4 cm per month or more but no positive correlation was apparent between the rate of growth as a single factor and yield within the limits of the observations, while the area with the slowest monthly growth had the highest tch per cm of cane. It is conceivable that a slower but steady growth could accumulate a greater mass per unit of length than one which is faster but subject to periodic physical checks. It is obvious therefore that while periodic linear measurements of cane growth are of great value in following the progress of a crop, they *by themselves* do not necessarily provide a complete basis of yield forecasting and require to be read in conjunction with other factors contributing to cane yield.

## Interference in the growth curve

It has been pointed out that the smoothness of a curve may be interfered with by factors operating in a manner prejudicial to the maintenance of an anticipated pattern of growth. For example, the break in the downward slope of the curve at Sucoma regularly occurring in different degrees between February and March is suspected of being associated with the period when lodging commences, and also possibly pre-flowering growth. It is noticed, however, that in linear growth a recovery occurs later. It has been found also that breaks in the upward slope, before the point of inflection, have occurred due to an interruption of the water supply for irrigation, caused either by canal breakages or an interrupted electricity supply for pumping. When this has occurred a reduction in growth rate has occurred, and no recovery has followed. A reduction in length of younger cane has been maintained until growth has ceased at the end of the growing period. It would appear then that the ultimate result of a check on growth depends not only on its extent but on the part of the growth curve on which it occurs, being most severe if it occurs prior to the point of inflection. At this stage of an increasing increase in growth, the crop is going through a development stage characterised by increasing canopy area and rapid stem elongation, whereas after the inflection point the canopy in extent has become more stabilised, and accompanied by a natural reduction in the rate of increase in length. A further point is of interest, however, in a February-March portion of the curve. Although linear growth has been temporarily reduced and a recovery occurs towards the anticipated final length, the query posed is whether or not any effect on the crop has occurred. No factual data are yet available, but it is considered that a loss in mass may have occurred and been responsible, at least in part, for the reduction in the tch per cm of cane index referred to earlier. This raises the question of the effect on general crop development of interruption during growth.

## Compensatory growth

It is commonly recognised that sugarcane has the ability to recover vegetatively from checks in development by the production of new growth of a compensatory nature. It has been shown, however, in a paper by Bull and Tovey[1] that the conception of recovery may be illusionary, since dry matter lost during the affected period may not be recouped and an ultimate reduction in total dry matter experienced, despite vegetative recovery. It was also shown that the moisture stress during the early stages of growth resulted in a drastic decline in final yield compared to that caused by a late season stress. In this category of growth the ultimate effect of recovery in lodged cane may be considered. It is obvious by examination of canes which have become erect after being recumbent, that considerable cellular activity has taken place in the growth rings in the affected region of the stems, an activity which could not have occurred without some utilisation of the elaborated products of photosynthesis. In other words the eventual attainment of an erect position and further linear growth could be accompanied by a reduction in dry matter which would not be regained.

## Discussion

A study of the different development phases of a crop of sugarcane in relation to its graphical representation of the mathematical interpretation of the latter has value in highlighting some important features in bringing a crop from planting to maturity. The quantitative data collected expressed as a growth curve emphasises for one thing the importance of early and quick growth during establishment, starting with the selection of seed cane with a high germination energy. Quick germination is followed by a rapid tillering phase leading into the quick initiation of the phase of stem elongation. This latter, leading up to the point of inflection in the growth should be rapid and without a break. This growth period appears to be of special importance on the ultimate behaviour of the crop and emphasises the necessity for adequate nutrition and water, and the absence of interfering factors such as weed competition. The position of the point of inflection — the half-way stage — in the development curve appears to be of special significance. It is the point at which rapid growth changes to slower growth, and is the key point in determining the potential ultimate growth reached at the time of finality. It is thus not only of importance in the time scale, but is also important with regard to its magnitude in the growth scale. It is of significance that factors causing a check in growth before the inflection point appear from present evidence to have a greater effect on crop development than if they appeared after the point of inflection.

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