SIMULATION OF EFFECT OF DIFFERENT VALUES OF OPERATING VARIABLES IN A CONTINUOUS PAN

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Abstract

A computerised steady-state model, Continuously Operating Raw Pan Simulation and Evaluation (CORPSE), is used to simulate the interaction of process variables on a continuous A-pan. A set of 3 simplified equations, based on the fundamental principles of operation of the pan, is derived. They express the relationship between the 5 main controllable operating variables, namely seed rate, product massecuite rate, seed crystal average size, product crystal average size and calandria steam temperature. This means that any 2, but only 2, of these variables can be fixed at arbitrarily desired values, and the remaining 3 will be defined in terms of the 3 equations. The effects of different selections of compartments on dilution water, as opposed to syrup feed, and of varying the crystal content of the product massecuite, are also simulated and discussed.

Introduction

It is useful to know how the different operating variables of a continuous pan are inter-related. Examples are:

(a) The effect of a change in one operating variable on the various output values.

(b) At what values the controllable operating variables must be run to achieve a desired pan performance.

It is usually not feasible to experiment with an operational pan because of:

The time required to reach the new steady-state operation; the amount of supervisory and analytical effort required; the influence of other process variations ("noise"); and the general constraints of factory production.

For this reason, a computerised mathematical model of a pan, called Continuously Operating Raw Pan Simultation and Evaluation (CORPSE), was written to simulate whatever operating conditions are specified.

CORPSE is a steady-state simulation, ie it does not simulate the transient response to any changes in operating conditions.

This paper describes the application of simulations to an A-sugar pan. It will not describe the operation of the computer program, which is dealt with in a previous publication.[1]

The approach was to take a Base Case simulation, representing typical operational conditions, and then to change one or more of the base case operating conditions in each test simulation.

Notation Conventions

- The names of operating variables will be written in capitals, eg SUPERSATN.
- Where a variable name consists of more than one word, eg MSC RATIO, the words will be linked by a break or underline character to avoid confusion, eg MSC_RATIO.
- The suffix _in will be used to refer to the seed or feed stream entering the first compartment, and the suffix _out to refer to the product or massecuite stream leaving the last compartment.
- An asterisk (*) will be used for a multiplication sign.
- The prefix Δ will be used to signify a difference, eg ΔMSC = MSC_out - MSC_in.

Figure 1. shows a diagrammatic representation of a continuous pan, together with the names of the variables of interest. Note that either dilution water or syrup can be fed into a compartment, but not both.

Underlying principles

1. The number of crystals/hour N_XTALS is assumed to be fixed in its passage from entry in the seed stream to exit in the massecuite stream, i.e. there is no dissolution of crystals nor formation of false grain or breakage into smaller pieces.

2. Crystals are all of a constant shape, although not necessarily of the same size. If characteristic size L refers to the length (mm) of a given side of a crystal, the volume of the crystal will then be

SHAPE_FACTOR * L^3,

where SHAPE_FACTOR = 1,0 for cubic crystals
= 2,0 for 1 × 1 × 2 crystals, with L referring to a short side, etc.

For the purposes of this simulation, the crystals were assumed to be cubic in shape.

In practice, one has to work with mean crystal sizes, rather than the size L of an individual crystal. The following concepts of mean crystal sizes will be used:

- The mean-size-by-number, as determined by measuring individual crystal sizes under a microscope grid and counting the frequency of occurrence, will be denoted by XSIZE.
- The mean-size-by-mass, as determined by sieving and weighing the different size fractions, is commonly known as the mean aperture, and will be denoted by MA.

3. Inside each compartment, there is the following mass transfer equation, per Wright[2]:

Increase in crystal size = ΔL
= K_GROWTH * (SUPERSATN − 1.0) * RET_TIME * IMPUR_EFFECT

where

K_GROWTH	=	Growth rate parameter, mm/h.
SUPERSATN	=	Supersaturation of the mother liquor (Saturation = 1,00)
RET_TIME	=	Retention time in a compartment (hrs), which is directly proportional to the compartment volume and inversely proportional to the massecuite rate through it.
IMPUR_EFFECT	=	Effect of impurity/water ratio, according to a function evaluated by Wright[2]
	=	exp (−1,75 * (Impurity/water ratio))

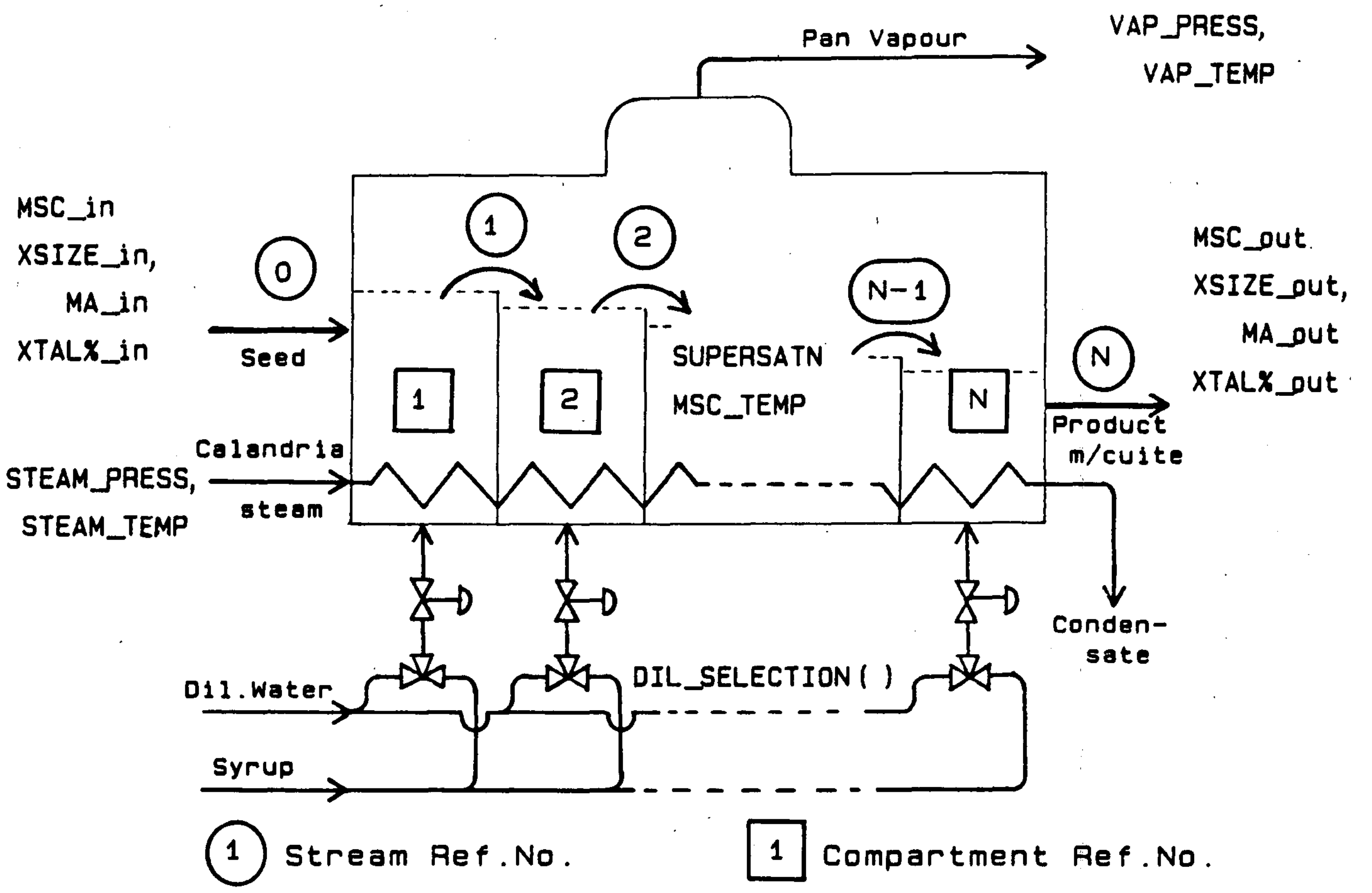

FIGURE 1 Representation of pan, showing main process variables of interest

The higher the impurity/water ratio, the lower will be the value of IMPUR_EFFECT and thus the lower will be the increase in crystal size.

4. For each compartment, the following heat transfer takes place: Water evaporated * Latent heat

= HTC * AREA * (STEAM_TEMP – MSC_TEMP)

where:

HTC	=	Heat transfer coefficient, kW/(m²°C)
AREA	=	Heat transfer area per compartment, m².
STEAM_TEMP	=	Temperature of heating steam in calandria, °C, at pressure STEAM_PRESS, kPa abs.
MSC_TEMP	=	Massecuite temperature, °C.
	=	VAP_TEMP + BPE
VAP_TEMP	=	Temperature of vapour from boiling massecuite, °C, at pressure VAP_PRESS, kPa abs.
BPE	=	Boiling Point Elevation of the massecuite, in °C, as determined by an empirical equation in CORPSE developed by Hoekstra[1].

Operating values to be considered as constant

Refer to Figure 1.

Physical aspects:

These refer to the construction of the pan used at Maidstone, and are considered as fixed for the purposes of this paper. They are:

- No. of compartments = 12
- Volume per compartment = 9,7 m³
- Heat transfer area per compartment = 91 m²

Process Parameters

These were determined by feeding factory measurements of the operating plant into CORPSE and running it in the mode for evaluating parameters. Typical values, which will be used here, are:

- Growth rate parameter	=	K_GROWTH
	=	1,60 mm/h
- Heat transfer rate parameter	=	HTC
	=	0,295 kW/m²°C for compartment No. 1, going down to 0,185 kW/m²°C for compartment No. 12.

Operating conditions

Certain of the operating values will be considered as fixed, either because the operator has to accept them as they come, or because they would not deliberately be varied for the sake of controlling the process.

This includes the following, at the Base Case values for which the simulations will be run:

- Solids % total mass of seed stream	=	BX_in
	=	86,0%
- Purity % of seed stream	=	PURITY_in
	=	86,0%
- Solids % total mass of syrup	=	SYRUP_BX
	=	67,6%

- Purity of syrup	= SYRUP_PUR = 83,7%
- Vapour pressure of pan	= VAP_PRESS = 14 kPa abs.
(This is equivalent to VAP_TEMP	= 52,6°C)

Operating values which can be varied

Directly and indirectly controllable variables

Some of the operating variables, mainly on the input side of the process, can be controlled directly, whereas others, on the output side, can only be varied indirectly, through the correct choice of settings of the directly controllable variables.

The list of controllable variables, used in the simulations, at their base case values, is:

Directly controllable

- Mass rate of massecuite in	= MSC_in = 20,0 tons/h
- Mean crystal size in (in terms of mean-size-by-mass)	= MA_in = 0,400 mm
(This is equivalent to mean-size-by-number	= XSIZE_in = 0,244 mm)
- Crystal % massecuite in	= XTAL%_in = 39,5%
- Pressure of calandria steam	= STEAM_PRESS = 98,7kPa abs.
(This is equivalent to STEAM_TEMP	= 99,3°C)
- Supersaturation	= SUPERSATN = 1,14 for compartment No. 1, increasing to 1,20 for compartment No. 12
- Selection of compart ments on dilution water, as opposed to syrup feeds	= DIL_SELECTION = (1,6,10,11,12)

Indirectly controllable

- Mass rate of massecuite out (in terms of mean-size-by-mass)	= MSC_out = 50,2 tons/h
Mean crystal size out	= MA_out = 0,528 mm
(This is equivalent to mean-size-by-number	= XSIZE_out = 0,385 mm)
- Crystal % massecuite out	= XTAL%_out = 49,0%

If SUPERSATN and XTAL%_out have been specified, it will automatically determine the values of solids % product massecuite and pan exhaustion, these being BX_out = 90,8% and EXHAUSTION = 63,8% for the Base Case.

Degrees of freedom

From the total list of 9 directly and indirectly controllable variables, the operator is free to fix any 6 of these at such values as he desires. The simplest is to fix the 6 directly controllable variables at arbitrary or convenient levels, and let the indirectly controllable variables find their own levels in the process.

From a practical point of view, it is desirable to aim at achieving specified values on certain of the indirectly controllable variables. For example, to achieve adequate exhaustion, the pan would have to be run in such a way that the crystal content in the product stream will be at a specified value, say XTAL%_out = 49%.

Because this indirectly controllable variable has been fixed, one of the directly controllable variables, say STEAM_PRESS, will no longer be free, but will have to be set at the appropriate value which takes into account the chosen values of the 1 indirectly and the remaining 5 directly controllable variables.

Figure 2(a) and 2(b) show the input and output print-outs of CORPSE for the Base Case, which represents the above situation. The simulation shows that the correct value to operate the steam pressure is at STEAM_PRESS = 98,7 kPa abs.

Further remarks on controllable variables

Average supersaturation

SUPERSATN is, in theory, a directly controllable variable. In practice, the operator usually strives to control it at as high a value possible without incurring spontaneous nucleation, so that, effectively, SUPERSATN does not offer much scope for controlling the performance of the pan, and will be considered as fixed for the purposes of the simulations.

Dilution water or syrup feed per compartment

Once having made his DIL_SELECTION choice, the user does not have any control over the amount of dilution water or syrup, as the case may be, which is fed into the individual compartment, because it is automatically regulated to keep the mother liquor at the desired level of SUPERSATN.

Illustration of fundamental relationships

Table 1 shows the values for a series of simulations, whereby the values of directly controllable variables MA_in and MSC_in were varied from the Base Case. For all the runs, the following Base Case values were maintained:

Crystal contents were XTAL%_in = 39,5% and XTAL%_out = 49,0%; compartments 1, 6, 10, 11 and 12 were on dilution water, and SUPERSATN ranged from 1,14 to 1,20 from the 1st to the last compartment. In each run, the program calculated the value of STEAM_TEMP and thereby STEAM_PRESS at which the pan would have to be run to achieve the desired value of XTAL%_out. Other output values of interest, MA_out and MSC_out, are also shown in Table 1.

Relationship between mean crystal size and massecuite rate

Figure 3 shows a plot of MA_out/MA_in = MA_RATIO vs. MSC_out/MSC_in = MSC_RATIO, which is a curved relationship.

Plotting MA_RATIO^3 vs. MSC_RATIO gives a straight line.
The explanation is as follows:
The mass of a single crystal is given by:
Crystal mass = Density * L^3 * SHAPE_FACTOR,
where L = characteristic size of that crystal
and SHAPE_FACTOR = 1, for the assumed cubic shape.

From Principle 1, the total number of crystals/hour N_XTALS is considered to remain constant in its passage through a pan. It follows that, at any point in the pan

Total crystal mass = N_XTALS * SPREAD_FACTOR * DENSITY * (XSIZE)^3 * SHAPE_FACTOR,

where XSIZE refers to the mean characteristic size on a number basis of all the individual crystals of typical size L, and SPREAD_FACTOR refers to the mathematical fact that, when there is a range of crystal sizes, the crystal of mean-volume-by-number will be slightly larger than the crystal of mean-size-by-number.

Continuously Operating Raw Pan Simulation and Evaluation (C.O.R.P.S.E.) DATE:07APR86 TIME: 17:05:44

RUN: X20_400 MILL: EXAMPLE MILL OBJECT:SIMULATIONS: BASE CASE: MSC_IN=20 TONS/H, MA_IN=.400 MM. STEAM_PRESS TO BE DET.

SUMMARY OF INPUT DATA.

O V E R A L L P A N .

NO. OF COMPARTMENTS (N): 12 TYPE OF PAN: A ASSUMED GRAIN SIZE DISTR.: ROSIN-RAMMLER PAN EXHAUSTN:

MOLASSES OR SYRUP FEEDS: SOLIDS % SOLUTION : 67.6 PURITY (%) : 83.7 MASS (TONS/HR):
HEATING STEAM TO PAN : PRESSURE(kPa ABS.): TEMPERATURE(DEG.C): MASS (TONS/HR):
VAPOUR ABOVE PAN : PRESSURE(kPa ABS.): 14.0 TEMPERATURE(DEG.C): 52.6 MASS (TONS/HR):

PARAMETERS OF PAN: NO.OF XTALS (*10**9) : EMPIRICAL EQUATIONS: HTC(J) = -0.010 *(J-6) + 0.245
GROWTH (K_GROWTH),MM/H: 1.60000 SUPERSATN(J) = 0.005 *(J-6) + 1.170
DISTR_PARAM(J) = 0.060 *(J-6) + 2.450

M A S S E C U I T E S T R E A M S .

STREAM REF.NO.:	0	1	2	3	4	5	6	7	8	9	10	11	12
MASS RATES(TONS/HR):													
TOTAL MASS :	20.00												
SOLIDS :													
SUCROSE(INCL.XTAL:													
WATER :													
XTAL :													
VOL.FLOWRATE,CU.M/H:													
XTAL SIZES (MM) :													
BY MASS FREQ: MA :	.400												
BY NR. FREQ: L1 :	.244												
MASSECU.PROPERTIES :													
SOLIDS%MASS<BRIX>:	86.00												
SUCROSE%MASS<POL>:													
PURITY (%) :	86.00												
XTAL % MASSECUITE:	39.50												49.00
DENSITY (GM/CC) :													
MOLASSES PROPERTIES:													
SOLIDS%MASS<BRIX>:													
SUCROSE%MASS<POL>:													
PURITY % <NUTSCH>:													
SUPERSATN (FRACN):													
IMPUR/WATER RATIO:													

C O M P A R T M E N T S .

COMPARTMENT REF.NO.	1	2	3	4	5	6	7	8	9	10	11	12
SPACE DETAILS :												
VOLUME (CU.M.) :	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70
RETENTION (HRS.) :												
CONTENTS PROPERTIES:												
GROWTH RATE(MM/H):												
MSC TEMP.(DEG.C.):												
MASS RATES(TONS/HR):												
SYRUP FEEDS :	0.00					0.00				0.00	0.00	0.00
MOVEMENT WATER :		0.00	0.00	0.00	0.00		0.00	0.00	0.00			
EVAPORATION :												
HEAT TRANSFER :												
HEAT TR.AREA,SQ.M:	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0
H.T.C.,KW/SQ.M-C :												
HEAT TR., KW :												

FIGURE 2(a) Input to Simulation Program (Base Case)

Continuously Operating Raw Pan Simulation and Evaluation (C.O.R.P.S.E.) DATE:07APR86 TIME: 17:13:15

RUN: X20_400 MILL: EXAMPLE MILL OBJECT:SIMULATIONS: BASE CASE: MSC_IN=20 TONS/H, MA_IN=.400 MM. STEAM_PRESS TO BE DET.

RESULTS OF RUN. ITERATION 3 OF 3 UNKNOWNS = 54, RELATIONSHIPS = 54

O V E R A L L P A N .

NO. OF COMPARTMENTS (N): 12 TYPE OF PAN: A ASSUMED GRAIN SIZE DISTR.: ROSIN-RAMMLER PAN EXHAUSTN: 63.8

MOLASSES OR SYRUP FEEDS:	SOLIDS % SOLUTION :	67.6	PURITY (%) :	83.7	MASS (TONS/HR):	42.0
HEATING STEAM TO PAN :	PRESSURE(kPa ABS.):	98.7	TEMPERATURE(DEG.C):	99.3	MASS (TONS/HR):	16.2
VAPOUR ABOVE PAN :	PRESSURE(kPa ABS.):	14.0	TEMPERATURE(DEG.C):	52.6	MASS (TONS/HR):	15.3

PARAMETERS OF PAN: NO.OF XTALS (*10**9) : 192.49 EMPIRICAL EQUATIONS: HTC(J) = -0.010 *(J-6) + 0.245
GROWTH (K_GROWTH),MM/H: 1.60000 SUPERSATN(J) = 0.005 *(J-6) + 1.170
DISTR_PARAM(J) = 0.060 *(J-6) + 2.450

M A S S E C U I T E S T R E A M S .

STREAM REF.NO.:	0	1	2	3	4	5	6	7	8	9	10	11	12
MASS RATES(TONS/HR):													
TOTAL MASS :	20.00	19.01	24.60	29.90	34.91	39.65	39.22	43.57	47.58	51.28	50.89	50.54	50.23
SOLIDS :	17.20	17.20	22.00	26.57	30.92	35.04	35.04	38.83	42.35	45.62	45.62	45.62	45.62
SUCROSE(INCL.XTAL:	14.79	14.79	18.81	22.64	26.28	29.73	29.73	32.89	35.84	38.58	38.58	38.58	38.58
WATER :	2.80	1.81	2.60	3.32	3.99	4.61	4.18	4.74	5.24	5.66	5.27	4.92	4.61
XTAL :	7.90	9.47	11.10	12.70	14.26	15.79	17.15	18.53	19.89	21.25	22.49	23.61	24.61
VOL.FLOWRATE,CU.M/H:	13.88	12.89	16.77	20.43	23.90	27.17	26.74	29.74	32.50	35.04	34.65	34.29	33.98
XTAL SIZES (MM) :													
BY MASS FREQ: MA :	.400	.421	.438	.454	.467	.479	.488	.497	.505	.513	.519	.524	.528
BY NR. FREQ: L1 :	.244	.261	.278	.293	.306	.319	.330	.341	.351	.360	.369	.377	.384
MASSECU.PROPERTIES :													
SOLIDS%MASS<BRIX>:	86.00	90.46	89.43	88.88	88.56	88.38	89.35	89.12	89.00	88.95	89.64	90.27	90.83
SUCROSE%MASS<POL>:	73.96	77.80	76.46	75.72	75.26	74.97	75.80	75.50	75.32	75.23	75.81	76.34	76.81
PURITY (%) :	86.00	86.00	85.50	85.19	84.98	84.83	84.83	84.72	84.63	84.57	84.57	84.57	84.57
XTAL % MASSECUITE:	39.50	49.83	45.10	42.47	40.84	39.81	43.73	42.52	41.81	41.44	44.20	46.72	49.00
DENSITY (GM/CC) :	1.44	1.48	1.47	1.46	1.46	1.46	1.47	1.46	1.46	1.46	1.47	1.47	1.48
MOLASSES PROPERTIES:													
SOLIDS%MASS<BRIX>:	76.86	80.99	80.75	80.68	80.67	80.69	81.08	81.07	81.09	81.14	81.44	81.73	82.02
SUCROSE%MASS<POL>:	56.96	55.75	57.13	57.80	58.18	58.41	56.99	57.37	57.59	57.70	56.64	55.58	54.53
PURITY % <NUTSCH>:	74.11	68.83	70.74	71.64	72.12	72.39	70.29	70.77	71.02	71.11	69.56	68.01	66.49
SUPERSATN (FRACN):	.94	1.14	1.15	1.15	1.16	1.16	1.17	1.17	1.18	1.18	1.19	1.19	1.20
IMPUR/WATER RATIO:	.86	1.33	1.23	1.18	1.16	1.15	1.27	1.25	1.24	1.24	1.34	1.43	1.53

C O M P A R T M E N T S .

COMPARTMENT REF.NO.	1	2	3	4	5	6	7	8	9	10	11	12
SPACE DETAILS :												
VOLUME (CU.M.) :	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70	9.70
RETENTION (HRS.) :	.753	.578	.475	.406	.357	.363	.326	.298	.277	.280	.283	.285
CONTENTS PROPERTIES:												
GROWTH RATE(MM/H):	.023	.027	.030	.033	.034	.029	.031	.033	.034	.030	.026	.023
MSC TEMP.(DEG.C.):	60.7	60.5	60.4	60.4	60.4	60.6	60.6	60.6	60.6	60.8	61.0	61.3
MASS RATES(TONS/HR):												
SYRUP FEEDS :		7.11	6.76	6.43	6.10		5.59	5.21	4.84			
MOVEMENT WATER :	.58					.87				.69	.67	.65
EVAPORATION :	1.56	1.52	1.47	1.41	1.36	1.30	1.25	1.19	1.14	1.08	1.02	.96
HEAT TRANSFER :												
HEAT TR.AREA,SQ.M:	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0	91.0
H.T.C.,KW/SQ.M-C :	.295	.285	.275	.265	.255	.245	.235	.225	.215	.205	.195	.185
HEAT TR., KW :	1036.1	1005.9	972.4	937.7	902.4	861.2	826.7	791.5	756.0	717.1	678.3	639.9

FIGURE 2(b) Output from Simulation Program (Base Case)

Table 1
Simulations to illustrate fundamental relationships

Run	X20_300	X20_400*	X20_500	X15_300	X15_400	X15_500	X25_400
Controllable Variables							
MSC_in (tons/h)	20,0	20,0	20,0	15,0	15,0	15,0	25,0
MA_in (mm)	0,300	0,400	0,500	0,300	0,400	0,500	0,400
STEAM_TEMP (°C)	110,5	99,3	92,1	108,1	97,7	91,1	100,2
MSC_out (tons/h)	59,5	50,2	44,3	52,9	44,4	38,9	55,6
MA_out (mm)	0,419	0,528	0,632	0,443	0,557	0,667	0,507
Parameters of Simplified Relationships							
K1	1,09	1,09	1,09	1,09	1,09	1,09	1,09
K2	9,46	8,99	8,49	9,71	9,33	9,00	8,62
K3	1,26	1,28	1,29	1,25	1,26	1,27	1,29
* = Base Case							
Note: In all cases, MSC_TEMP = 60,7°C.							

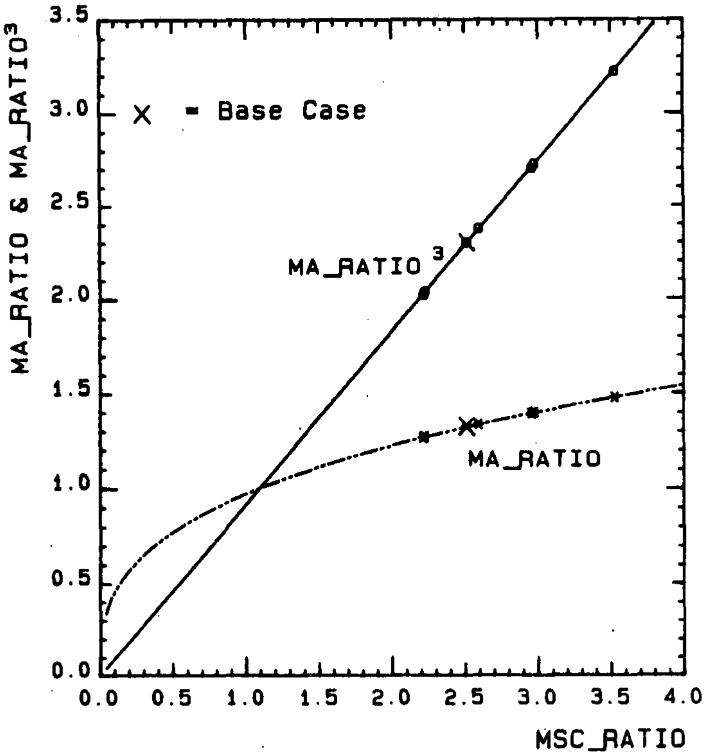

FIGURE 3 Relationship between massecuite ratio and crystal ratio

More specifically, SPREAD_FACTOR =

$$\frac{\text{Mean-volume-by-number}}{(\text{Mean-size-by-number})^3}$$

Appendix 1 shows the calculation of SPREAD_FACTOR. In these simulations, SPREAD_FACTOR_in = 1,829 and SPREAD_FACTOR_out = 1,459.

Referring to the _in and _out streams, we have the proportionality:

$$\frac{\text{Total Crystal mass out}}{\text{Total Crystal mass in}}$$

$$= \frac{\text{XTAL\_out}}{\text{XTAL\_in}}$$

$$= \frac{\text{SPREAD\_FACTOR\_out}}{\text{SPREAD\_FACTOR\_in}} * \frac{\text{XSIZE\_out}^3}{\text{XSIZE\_in}^3},$$

because N_XTALS, DENSITY AND SHAPE_FACTOR remain constant and cancel out.

For A-pans, determinations of mean sizes are more easily done by sieving than by counting under a microscope, and for this reason the mean-size-by-mass or mean aperture MA is a more convenient value than mean-size-by-number XSIZE. The concept of MEANS_FACTOR $= \frac{\text{MA}}{\text{XSIZE}} \times$ is introduced.

Appendix 1 shows the calculation of MEANS_FACTOR. In these simulations, MEANS_FACTOR_in = 1,642 and MEANS_FACTOR_out = 1,375.

The crystal contents (mass crystal % total mass) = XTAL%_in and XTAL%_out are fixed for all runs, which fixes the relationship between massecuite rate ratio and crystal mass ratio.

We have

MSC_RATIO

$$= \frac{\text{MSC\_out}}{\text{MSC\_in}}$$

$$= \frac{\text{XTAL\%\_in}}{\text{XTAL\%\_out}} * \frac{\text{XTAL\_out}}{\text{XTAL\_in}}$$

$$= \frac{\text{XTAL\%\_in}}{\text{XTAL\%\_out}} * \frac{\text{SPREAD\_FACTOR\_out}}{\text{SPREAD\_FACTOR\_in}} * \frac{\text{XSIZE\_out}^3}{\text{XSIZE\_in}^3}$$

$$= \frac{\text{XTAL\%\_in}}{\text{XTAL\%\_out}} * \frac{\text{SPREAD\_FACTOR\_out}}{\text{SPREAD\_FACTOR\_in}} * \frac{\text{MEANS\_FACTOR\_in}^3}{\text{MEANS\_FACTOR\_out}^3} * \frac{\text{MA\_out}^3}{\text{MA\_in}^3}$$

$$= \frac{39{,}5}{49{,}0} * \frac{1{,}459}{1{,}829} * \frac{1{,}642^3}{1{,}375^3} * \frac{\text{MA\_out}^3}{\text{MA\_in}^3}$$

$= 1{,}09 * (\text{MA\_RATIO})^3$

more generally,

$$\frac{\text{MSC\_out}}{\text{MSC\_in}} = \text{K1} * \frac{\text{MA\_out}^3}{\text{MA\_in}^3} \qquad \text{. . . . . (1)}$$

The above workings assume that the same ratios of SPREAD_FACTOR_out / SPREAD_FACTOR_in and MEANS_FACTOR_in/MEANS_FACTOR_out hold for all combinations of MA_out and MA_in, which is not strictly true, but to a large extent these 2 ratios cancel each other out, so that the above cubic relationship (1) will be substantially correct.

It should be noted that this relationship (1) between MA_RATIO and MSC_RATIO is independent of the pan compartment sizes and heat transfer areas.

Relationship between massescuite flow rates and crystal growth

If the massecuite flow rate through the pan is increased, the retention time for the pan as a whole will decrease in inverse proportion.

From Principle 3.:

$\Delta XSIZE \propto RET\_TIME$

$$\text{approx.} \propto \frac{1}{\frac{1}{2}(MSC\_in + MSC\_out)}$$

Figure 4 shows a plot from the simulating runs data of XSIZE_out-XSIZE_in = ΔXSIZE vs. 1/(MSC_in + MSC_out).

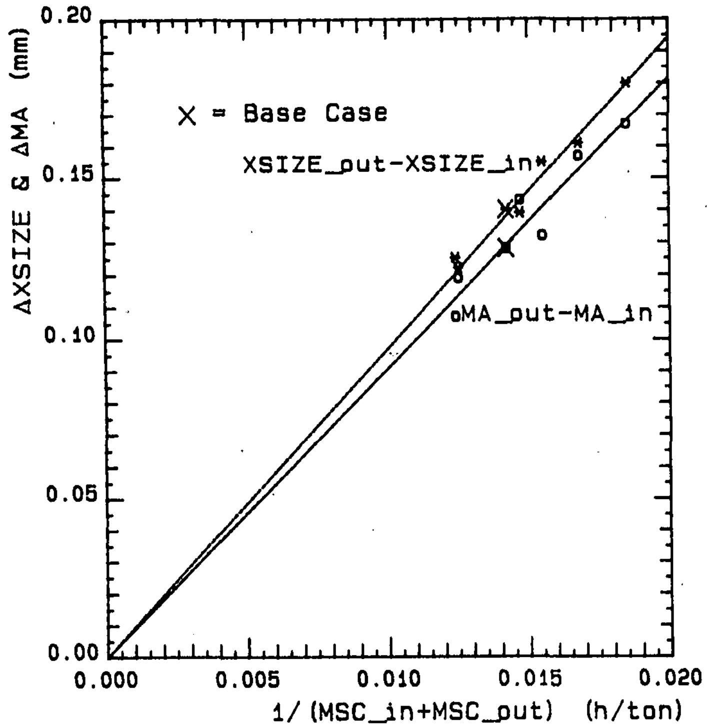

FIGURE 4 Relationship between massecuite flow rates and crystal growth

The result is close to a straight line, the deviations being mainly due to the retention time not being exactly inversely proportional to the arithmetic mean of the massecuite in and outflow rates.

As discussed earlier, mean aperture is a more useful measure of mean crystal size than the above XSIZE = mean-size-by-number.

In Fig. 4, a plot also is shown of MA_out − MA_in = ΔMA vs. 1/(MSC_in + MSC_out).

Because of the already discussed variations in MEANS_FACTOR = MA/XSIZE, this plot shows larger deviations from a straight line than the previous.

However, for the purposes of deriving simplified interrelationship of process variables, it is sufficiently close to consider it as a proportionality of

$$MA\_out - MA\_in = \frac{K2}{(MSC\_in + MSC\_out)} \quad \ldots\ldots\ldots\ldots (2)$$

The values of K2 calculated from the simulation runs are shown in Table 1, with an average of K2 = 9,08.

The value of parameter K2 of an individual pan will depend mainly on the effective volume of its compartments.

Relationship between calandria steam temperature and massecuite flow rates

Although it is more usual to refer to and control the calandria steam in terms of its pressure, for the sake of explaining its relationship to the process it is more convenient to refer to its saturated temperature, STEAM_TEMP, at the particular pressure STEAM_PRESS.

Because the object of the heating steam is to transfer heat into the massecuite to evaporate water from it, it is logical to think in terms of its driving force, which is the difference between the steam and the massecuite temperatures.

Figure 5 shows a plot from the simulation runs of STEAM_TEMP − MSC_TEMP vs. the difference in massecuite flow rates out and in, ie vs. MSC_out − MSC_in = ΔMSC.

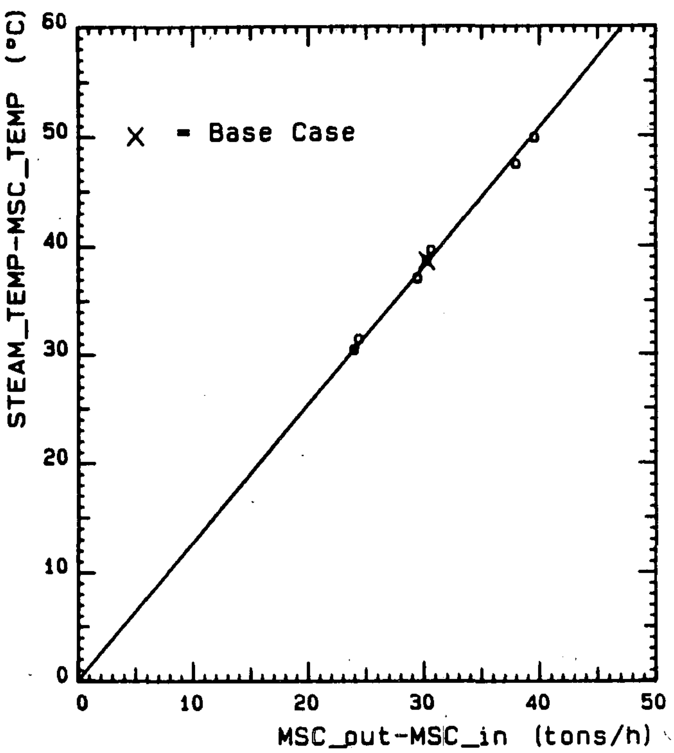

FIGURE 5 Relationship between calandria steam temperature and massecuite flow rates

It is seen that the points lie on a straight line, passing through the origin.

In the explanation which follows, please note that:

- For brevity, solids % mass (BX) values will be taken as fractions instead of percentages.

- The solids rate in TOT_SYRUP_in is equal to the difference in the respective solids rates in MSC_out and MSC_in.
- For a given syrup/water configuration of DIL_SELECTION, the amount of TOT_DIL_in will be approximately proportional to TOT_SYRUP_in, by a ratio DIL_FACTOR.

From Principle 4, the heating steam has to evaporate sufficient of the water in the MSC_in, TOT_SYRUP_in and TOT_DIL_in streams entering the pan to achieve the desired water content of the MSC_out product stream.

Water entering in MSC_in seed stream

= MSC_in * (1 − BX_in)

Water entering in TOT_SYRUP_in and TOT_DIL_in

= (MSC_out * BX_out − MSC_in * BX_in)
* (1 − SYR_BX + DIL_FACTOR)/SYR_BX

Water leaving in MSC_out product stream
= MSC_out * (1 − BX_out)

Combining the above will give the amount of water evaporated.

Re-arranging:

Total water evaporated

= MSC_in * (BX_out − BX_in) * (1 + (1 − SYR_BX + DIL_FACTOR)/SYR_BX) + MSC_out − MSC_in) * (BX_out * (1 + (1 − SYR_BX + DIL_FACTOR)/SYR_BX)− 1)
= A1 * MSC_in + A2 * (MSC_out − MSC_in),

where A1 and A2 are approximately constant.

From Principle 3, the water evaporated will be proportional to (STEAM_TEMP − MSC_TEMP). Because VAP_PRESS and SUPERSATN are assumed to be fixed, it follows that MSC_TEMP must likewise be considered as fixed, at an average value of 60,7°C over the compartments.

This can written as:

(STEAM_TEMP − MSC_TEMP) * AREA * HTC/LAT_HEAT

= A1 * MSC_in + A2 (MSC_out − MSC_in), or

STEAM_TEMP − MSC_TEMP
= K4 * MSC_in + K3 * (MSC_out − MSC_in),
where K4 and K3 are constants.

This represents a series of parallel straight lines of common slope K3 and intercepts = K4 * MSC_in.

The average values are K3 = 1,21 and K4 = 0,09.

Because the effect of K4 on the rest of the equation is small, it will be ignored for the purposes of obtaining general relationships between the operating variables, thus giving

STEAM_TEMP − MSC_TEMP = K3 * (MSC_out − MSC_in) . . . . (3)

The effect of leaving K4 out is that K3 increases slightly.

The values of K3 for each of the simulations are given in Table 1, with an average value of K3 = 1,27.

The value of parameter K3 of an individual pan will depend largely on the size of heat transfer area and its heat transfer coefficient.

Use of the relationship equations

To summarise the 3 basic, simplified equations which were derived, are:

$$\frac{\text{MSC\_out}}{\text{MSC\_in}} = \text{K1} * \frac{\text{MA\_out}^3}{\text{MA\_in}^3} \qquad \ldots\ldots\ldots (1)$$

$$\text{MA\_out} - \text{MA\_in} = \frac{\text{K2}}{\text{MSC\_in} + \text{MSC\_out}} \qquad \ldots\ldots\ldots (2)$$

$$\text{STEAM\_TEMP} - \text{MSC\_TEMP} = \text{K3} * (\text{MSC\_out} - \text{MSC\_in}) \qquad \ldots\ldots\ldots (3)$$

For a specific pan, the values of the parameters K1, K2 and K3 can be determined from one or more previous runs of the pan, by simply substituting the measured values of MSC_out, MSC_in, etc. into the equations. The following values were obtained from operating pans:

Tests done on 6-compartment experimental A-pan at Amatikulu in August 1982:

Date	*K1*	*K2*	*K3*
3 August	1,24	13,7	1,05
4 August	1,12	13,3	1,44

Tests done on 12-compartment A-pan at Maidstone in January 1983

Date	*K1*	*K2*	*K3*
6 January	1.08	16.0	0.93
7 January	0.76	15.5	0.96
8 January	0.83	10.1	1.44
9 January	1.00	13.4	0.95
11 January	0.92	13.7	1.08
20 January	1.22	9.2	1.24
21 January	1.62	9.4	0.97
25 January	0.97	6.1	2.23
27 January	0.54	16.8	1.77

Although the variation in each parameter is fairly large, this is likely to be due to:

- Measurement problems, such as accurately determining massecuite flows and obtaining representative crystal samples.
- Operational problems, such as running at the desired level of supersaturation in each compartment, and deviations from steady-state operation.
- Variations in operational variables which had been considered as "fixed", such as brixes, purities and crystal contents of massecuite and syrup, and the syrup/dilution water selection. With improvements in automatic control of the process and better facilities for measuring flow rates, the parameter values obtained should be more consistent.

Because there are 5 variables in the 3 equations, namely MSC_out, MSC_in, MA_out, MA_in and STEAM_TEMP, the user can fix any 2 of them at whatever practical values he desires, and the remaining 3 can be determined by solving the equations.

Because the pan pressure VAP_PRESS and SUPERSATN are considered as fixed, MSC_TEMP will remain constant, and is therefore not one of the variables to be solved.

Depending on the choice of which 2 variables the user wants to fix, the equation (1) might end up by being reduced to having 1 unknown, but in a cubic-linear form. Although it cannot be solved algebraically, the left-hand and the right-hand sides can be graphically plotted as functions of the unknown, the solution of which is where the curves intersect.

Other effects

Choice of compartments on dilution water

For the Base Case, as well as all the foregoing simulations, compartments 1, 6, 10, 11 and 12 were put on dilution water, and the remainder on syrup. For each compartment, the control valve for admitting that particular feed material into the compartment would admit as much of the material as required to maintain the supersaturation in that compartment at the prescribed level.

By way of comparison, runs were made on the Base Case specifications, but using alternative selections of compartments on dilution water, namely: None; compartments 1, 2, 3 and 4; compartments 9, 10, 11 & 12.

Table 2 shows the results of interest. The reason for the particular DIL_SELECTION of the Base Case was simply that this combination was popularly used in the factory.

The Base Case selection of (1, 6, 10, 11, 12) shows a rise in crystal % total mass for each compartment which is on dilution water, and drops for compartments which are on syrup feed.

The case of no dilution water shows an immediate rise in crystal content in compartment 1, but which remains at roughly that level right up to compartment 12 to the specified final crystal content of 49 %. The simulation shows that the amount of syrup feed to compartment 1 is considerably less than for the other compartments, which receive a gradually reducing amount of syrup with successive compartments.

For "early" addition of water, ie DIL_SELECTION = (1, 2, 3, 4), the crystal content rises to quite high levels over compartments on water, and then gradually drops in all the subsequent compartments on syrup to reach the specified value of XTAL%_out = 49 %.

For "late" addition of water, ie DIL_SELECTION = (9, 10, 11, 12), the compartments 1-8 on syrup feed remain at comparatively low crystal contents, but which then rises in the last compartments which are on water.

Some other aspects of choice of compartments on dilution water must also be considered:

- Output rate and final crystal size:

In all cases, the input rate of massecuite MSC_in = 20 tons/hr and the mean-size-by-mass of seed is MA_in = 0,400 mm.

It can be seen that a policy of adding water in the later compartments, such as for the Base Case and DIL_SELECTION = (9, 10, 11, 12), gives a higher rate of output MSC_out. The simulations show these effects in Table 2.

In terms of the equation in Principle 3, these 2 effects work in opposite directions, but the latter effect pre-dominates slightly, thus allowing a higher rate of mass transfer. The converse happens when dilution water is fed into the earlier compartments.

The larger product crystal sizes which result from the addition of dilution water to the later compartments are a direct consequence of the abovementioned higher product massecuite rates and the proportionality between MSC_RATIO and MA_RATIO[3], per equation (1).

- Effect on heating steam:

As might be expected, the lowest consumption of heating steam occurs for the case of zero dilution water, which consequently also has the lowest steam pressure requirement. For the case of addition of water to the earlier compartments, the impurity/water ratio in these compartments already rises to a high level, thus reducing the rate of crystal growth and

Table 2

Effect of different combinations of compartments on dilution water

Case	Base Case		No Dil. Water		Early Water		Late Water	
Compartments on Dil. Water	(1, 6, 10, 11, 12)		(O)		(1, 2, 3, 4)		(9, 10, 11, 12)	
	Feed, t/h	Xtal%	Feed, t/h	Xtal%	Feed t/h	Xtal%	Feed, t/h	Xtal%
Seed		39,5*		39,5*		39,5*		39,5*
Compartment 1	0,58W	49,8	0,81S	48,4	0,31W	49,8	3,28S	44,4
2	7,11S	45,1	4,28S	48,2	0,89W	56,4	6,56S	42,3
3	6,76S	42,5	4,17S	47,8	0,94W	60,7	6,31S	40,8
4	6,43S	40,8	4,01S	47,5	0,96W	63,5	6,03S	39,9
5	6,10S	39,8	3,82S	47,4	5,44S	55,8	5,73S	39,3
6	0,87W	43,7	3,62S	47,3	4,80S	52,3	5,42S	38,9
7	5,59S	42,5	3,42S	47,4	4,39S	50,5	5,10S	38,8
8	5,21S	41,8	3,21S	47,5	4,05S	49,5	4,77S	38,8
9	4,84S	41,4	3,00S	47,8	3,76S	49,0	0,67W	41,7
10	0,69W	44,2	2,80S	48,1	3,49S	48,8	0,65W	44,4
11	0,67W	46,7	2,60S	48,5	3,23S	48,8	0,63W	46,8
12	0,65W	49,0*	2,40S	49,0*	2,98S	49,0*	0,62W	49,0*
MSC_in, t/h	20,0*		20,0*		20,0*		20,0*	
MSC_out, t/h	50,2		47,3		42,9		51,1	
MA_in, mm	0,400*		0,400*		0,400*		0,400*	
MA_out, mm	0,528		0,517		0,501		0,531	
STEAM_PRESS, kPa abs.	98,7		65,6		77,5		93,3	
STEAM_TEMP, °C	99,3		88,2		92,6		97,7	
Steam consumption, t/h	16,2		11,3		13,0		15,6	
Total Ret. time, h	4,68		5,29		6,68		4,34	
Av. Impurity/Water Ratio	1,28		1,40		1,59		1,23	

* = Fixed to same value as Base Case.
S = Compartment on Syrup
W = Compartment on Water

transfer of sucrose out of solution. To prevent the supersaturation level from being exceeded, additional dilution water is required, which in turns needs a higher steam pressure. The converse applies to the case of adding water to the later compartments.

Conclusion: There seems merit in reducing dilution water to the minimum, or else to apply it preferably only to the later compartments.

Effect of syrup brix

To see what happens if the pan is run at a different value of syrup brix (solids % solution) than the Base Case of SYRUP_BX = 67,6 %, a simulation was run at Base Case conditions, but with SYRUP_BX = 60,0 %.

The MSC_out and MA_out outputs were identical to those of the Base Case, as were also the respective impurity/water ratios in the individual compartments.

Contrary to expectation, the higher water content of the syrup did not cause a reduction in the amount of dilution water required, but an increase. The explanation is that, for those compartments which are on syrup feed, a larger amount of water has to be evaporated, thus necessitating a higher steam temperature and therefore steam pressure. This higher steam temperature will also apply to the compartments on dilution water, with consequent higher rate of heat transfer into them. More dilution water will have to be fed into them to prevent the mother liquor from being concentrated beyond the specified supersaturation.

Conclusion: Lower syrup brix requires a larger amount of dilution water.

Effect of changes in crystal content of product massecuite

In all the previous cases, it was assumed that the crystal content in the product stream would be fixed, ie XTAL%_out = 49,0 %. For this section the effect of running at a different value of XTAL%_out = 51,0% was simulated. Contrary to expectation that a higher STEAM_PRESS is required for higher evaporation and thus a higher crystal content, the pressure dropped from 98,7 to 89,4 kPa abs.

The explanation is that, to increase the crystal content (and the exhaustion, namely from 63,8 % to 66,0 %), less mother liquor should remain in the product. This means more sucrose has to be crystallised from the mother liquor, thus requiring less syrup feed. The effect of this is to increase the average impurity/water ratio, which will reduce the growth rate and thus the product tonnage of crystal, which will further reduce the amount of syrup required, from the Base Case rate of 42,0 tons/h to 38,0 tons/h.

The lower syrup feed will require less evaporation, and hence require a lower STEAM_TEMP and STEAM_PRESS.

Conclusion: Under conditions of a fixed seed feed specification and dilution water selection, the steam pressure must be reduced to increase the crystal content of the product massecuite, and vice versa.

For rough calculation of the change in STEAM_TEMP required, a revised MSC_out can be calculated by multiplying the original MSC_out by (Old XTAL%_out)/(New XTAL%_out), and using this in equation (3).

Conclsuion

Accurate interrelationships of the operating variables are best determined by using a computer simulation such as CORPSE. As an alternative, the equations (1) to (3) make it possible to determine qualitatively and, with some loss in accuracy, quantitatively, what the effect of changes in the main control variables of a continuous pan will be.

REFERENCES

1. Hoekstra, RG (1985). Program for simulating and evaluating a continuous A-pan. *Proc S Afr Sug Techn Assn* 59, p. 48-57.
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APPENDIX 1

Sugar Grain Size Relationships

Recent investigations have shown that the Rosin-Rammler probability density function (p.d.f.), applied to the size-frequency-by-number distribution, best fits A-pan sugar grain size distributions in the majority of cases. This is given by:

$$f(x) = \frac{p}{k} \cdot (x/k)^{p-1} \exp(-(x/k)^p)$$

where x = size (length), >0, mm.
p = shape parameter, >0, dimensionless
k = scale paramter, >0, with same dimensions as x
$\Gamma(p)$ = Gamma function with parameter p, defined as:
$\equiv \int_0^\infty t^{p-1} \exp(-t)\,dt$, for $p > 0$
$= (p-1)!$ for p a positive integer.

The corresponding cumulative distribution function (c.d.f.), which is used in determining the parameters p and k by fitting to experimental results of grain counting, is obtained by putting $t = (x/k)^p$ in the above equation and integrating:

Probability (Randomly selected grain size $\leqslant x$)

$$= F(x) = \int_0^{(x/k)^p} \exp(-t)\,dt$$

The mean-size-by-number is obtained from:

$$XSIZE = \int_0^\infty x\, f(x)dx = k\Gamma(1 + 1/n)$$

The p.d.f. for the size-frequency-by-mass (applied to sieving results) can be derived from f(x) as:

$$g(x) \equiv \frac{\int_0^x t^4 f(t)\,dt}{\int_0^\infty t^3 f(t)\,dt} = \frac{p.(x/k)^{p+2}}{k\Gamma(1 + 3/p)} \exp(-(x/k)^p)$$

and the corresponding c.d.f. as:

$$G(x) \equiv \frac{1}{r\,(1 + 3/p)} \int_0^{(x/k)^p} t^{3/p} \exp(-t)\,dt$$

The mean-size-by-mass is obtained from:

$$MA = \int_0^\infty x g(x)dx = \frac{k\Gamma(1 + 4/p)}{\Gamma(1 + 3/p)}$$

It can similarly also be shown that

$$\text{Mean-volume-by-number} = \int_0^\infty x^3 f(x)(dx) = k^3\Gamma(1 + 3/p)$$

Relationships referred to in the text are:

$$SPREAD\_FACTOR = \frac{\text{Mean-volume-by-number}}{(\text{Mean-size-by-number})^3} = \frac{\Gamma(1 + 3/p)}{(\Gamma(1 + 1/p))^3}$$

$$MEANS\_FACTOR = \frac{MA}{XSIZE} = \frac{\Gamma(1 + 4/p)}{\Gamma(1 + 3/p).\Gamma(1 + 1/p)}$$

Note that the scale parameter k cancels out, so that these relationships are functions of only the shape parameter p.