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SHAPE FACTORS OF BAGASSE PARTICLES

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Abstract

Details are presented on a method to determine shape factors for bagasse which was fractionated by mechanical sieving. The length, breadth and thickness of 200 particles selected randomly by means of a riffler from each of the size fractions were measured manually or by microscope, depending on size. From these measurements average values of the volume and surface shape factors for each fraction were calculated. The values obtained are used in the formula for the specific surface area of bagasse.

Introduction

Everyone who has worked with bagasse is aware of the tremendous range of particle sizes and shapes which is exhibited by this material. Bagasse particle size and shape have a profound influence on sugarcane processing operations such as diffusion, milling and mechanical handling as well as combustion in boilers and all information on these aspects should find application in understanding the behaviour of bagasse. Only one reported account of an investigation into particle shapes of bagasse could be located in the literature, namely that by Rein (1972, pp. 129-135) who indicated that further work on this topic should prove beneficial. The relative lack of information has led to this study being done as a step towards improved characterisation of bagasse.

The investigation described here was carried out using final dewatered bagasse (which was subsequently dried in a laboratory oven) from one particular factory which uses diffusion as the sugar extraction process. Because of the amount of work involved in evaluating bagasse shape factors it was not possible to carry out a similar study on other types of bagasse such as that from a milling tandem, first mill bagasse or shredded cane. It is suggested that a similar study be carried out on these materials to establish whether the results obtained have universal application or change significantly according to the method used to prepare and/or process the cane. It would also be beneficial to know to what extent cane variety influences the distribution of particle shape factors.

Bagasse particles fall into two broad categories of shape, namely long stringy fibres consisting of individual strands or conglomerates of parallel multiple strands, and short, roundish particles which are generally denoted by the name pith. The length of the fibrous particles can vary from about 1 mm to as much as 100 mm while their width can have a range from 0,3 mm to about 10 mm and the thickness from 0,1 mm to 5 mm. Pith particles exhibit minimum dimensions of approximately 0,05 mm and maximum dimensions of around 2-3 mm. The range of particle sizes of pith is therefore much less than that of the fibrous component.

Definition of Shape Factors

Sieving is a very convenient and relatively simple means of dividing bagasse particles into a number of size-related fractions. However, when a bagasse sample has been fractionated by sieving the aperture size of the sieve bears little relation to the actual dimensions of the particles except that a particle that passes through a sieve will have a width that is less than $\sqrt{2}$ times the aperture size (for square apertures). The length of the fibrous particles and their interlocking behaviour often contribute to their not passing through the sieve even though their width and thickness are much smaller than the aperture size.

In the case of a spherical particle only one dimension, namely its diameter, d, is needed to determine related data such as its volume or surface area. For example, the volume of a sphere is calculated by $\pi/6\ d^3$, and its surface area by $\pi\, d^2$. In sieve analysis the sieve aperture is used as the characteristic dimension (i.e. instead of 'd'), but to get any meaningful value of actual particle size, volume or specific surface area from bagasse sieving results it is necessary to find a relationship between the actual values (of volume or surface area etc.) and the sieve aperture. This relationship is usually expressed in terms of shape factors.

The volume V of an object can be expressed mathematically by the cube of a characteristic dimension X multiplied by an appropriate factor, i.e.

$$V = \alpha_v X^3 \qquad (1).$$

In this case X represents the sieve aperture and α_v is called the volume shape factor the determination of which will be covered below. In the case of a sphere α_v has the value of $\pi/6$.

In similar fashion the surface area S of an object is calculated from the square of a characteristic dimension X multiplied by an appropriate surface shape factor, α_s:

$$S = \alpha_s X^2 \qquad (2).$$

In principle, the determination of the shape factors for individual particles is done by establishing the actual volume, or surface area, and substituting these values in equations (1) or (2) respectively.

Nomenclature

Symbol	Meaning	Units
a_1,a_2,a_3	curve-fitting constants	--
d	diameter	m or mm
m_i	mass fraction	--
q	area correction factor	--
r	volume correction factor	--
s	specific surface area	mm^2/g
x_i	sieve aperture	mm or μm
A_p	projected area of particle	mm^2
L	length of particle	mm
S	surface area of particle	mm^2
T	thickness of particle	mm
V	volume of particle	mm^3
W	width of particle	mm
X	characteristic dimension of particle	m or mm
X_i	characteristic dimension, ith mass fraction	m or mm
α_s	surface shape factor	--
α_v	volume shape factor	--
ρ	particle density	g/mm^3

Materials and Methods

For the determination of the shape factors samples of oven dried bagasse that had been allowed to reach moisture equilibrium with the surrounding atmosphere were divided into size fractions using stainless steel test sieves of 200 mm diameter. The sieve apertures used were, in descending order: 6 700, 4 000, 2 800, 2 000, 1 400, 1 000, 850, 600, 425 and 300 μm. The sieving procedure has been documented (Bernhardt, 1993). Since there is a considerable range of particle shapes in bagasse it was decided to analyse a relatively large number of particles (200) from each size fraction and determine the volume and surface area of each particle as described below. From the calculated volume or surface area the respective shape factors for each particle were determined using equations (1) or (2). The X value used for the particles in any one fraction was the mean value between the aperture of the sieve that allowed that fraction to pass through, x_1, and that of the sieve that retained the fraction, x_2, (in other words, the mean of the oversize and undersize for that fraction). Since fractionation by mass was performed the appropriate formula for the determination of this mean is (Herdan, 1960, p. 33):

$$X_i = \sqrt[3]{\frac{(x_2^2 + x_1^2)(x_2 + x_1)}{4}} \qquad (3).$$

The average value of the respective shape factor was then determined for the 200 particles measured in each size fraction.

Determination of the actual volume and surface area of bagasse particles necessitated the adoption of a geometrical model for the particle shape to determine which measurements should be taken. Two possible models were considered, namely a cylindrical form and a rectangular block shape. A pilot determination of the volume and surface area of 20 particles in each of eight fractions was done in which approximation of bagasse particles to a cylindrical shape (which requires only two measurements, namely length and diameter) was compared with conformity to a rectangular block shape (requiring the determination of three dimensions, namely length, width and thickness). The rectangular block model was adopted because most of the particles that were measured exhibited widths which were significantly different from the thicknesses.

Heywood (1970) gave a very precise definition of length, width and thickness in terms of minimum distances between three mutually perpendicular pairs of parallel planes that are tangential to the particle, the reference plane being chosen as the plane of greatest stability from which the thickness is determined. These definitions are particularly appropriate in the case of microscopic analysis, although the determination of thickness does present specific difficulties. In the evaluation of the size and shape of bagasse particles it was found necessary to relax somewhat Heywood's stipulation that the width should be measured in a direction at right angles to that of the thickness. The reference direction was taken as the longest straight line distance exhibited by the particle, which for the fibrous particles was the longitudinal axis. The thickness was taken as the shortest dimension at right angles to the length and the width as the longest dimension parallel to the thickness after rotating the particle on its long axis. In other words, the width was not necessarily measured at right angles to the thickness.

It needs to be stated that whereas the rectangular block model seemed to be the most appropriate one covering the overwhelming majority of particles, some personal judgement was required with particles that consisted of several fibrous strands of similar or different length which adhered to one another in one part but were separated otherwise. Difficulty was also experienced in the analysis of microscopic pith particles many of which presented a roundish or oval shape and had a rather loose, fluffy appearance in contrast to the clear, distinct dimensions of the fibrous particles. It was frequently difficult to decide whether what was being measured represented one large irregularly shaped particle or a number of particles that had coalesced on the microscope slide. Furthermore, for these irregularly shaped particles, the selection of the dimensions of length, width and thickness was frequently a matter of subjective choice.

Two modes of particle measurement were employed. The larger particles (those retained by 600 μm and larger sieves) were measured manually using vernier callipers which could measure to the nearest 0,02 mm while the sizes of particles retained by the sub-600 μm sieves were measured with a Kontron image analyser which used a microscope camera magnification of 44 times. The latter measurements were recorded to the nearest μm. It was, of course, not possible to rotate particles on the Kontron microscope slide, so the thickness was taken as the shortest distance exhibited by the particle at right angles to the length, while the width was taken as the longest distance parallel to the thickness. The twisted fibres that were measured by hand were first straightened artificially before measurement while an estimate for the straight line length was made for those twisted fibres examined microscopically. It was found that the particles retained by the 600 μm sieve contained a large number of thin fibrous particles many of which were too long to be accurately measured on the Kontron, but also a significant proportion of small pith particles which were too small to be measured satisfactorily by the vernier callipers. It was therefore decided to measure 100 particles by hand and 100 particles by Kontron for that fraction. For the other fractions the one or the other method could be used for all 200 particles in that fraction.

To accommodate the deviation of the particle's shape from a strict rectangular block shape correction factors q and r were used. If a bagasse particle was resting on a horizontal surface (its plane of greatest stability) and its length was measured as L and width as W then the projected area A_p was calculated by equation (4)

$$A_p = q\,LW \qquad (4)$$

where $q < 1$. The factor q has been called the 'area ratio' by Heywood. The actual value of q that was used was 0,75. This is an assumed value that has been estimated by the author from reported figures by Heywood of 0,5-0,9 for prismoidal particles and 0,72-0,82 for rounded particles. It is postulated that the actual projected area of a bagasse fibre is 0,75 times that of a rectangular block of similar length and width. The volume V of the particle was corrected by the factor r (again < 1) and the formula used, in terms of its length L, width W and thickness T is

$$V = qr\,LWT \qquad (5).$$

The name given to r by Heywood is 'prismoidal ratio' and the value that was used is again 0,75. The range of values reported by Heywood for prismoidal particles is 0,53-0,9 and for rounded particles 0,52-0,75. It is therefore estimated that, on average, the volume of actual bagasse particles is 0,75 × 0,75 times (i.e. 0,563 times) that of a rectangular block of the equivalent dimensions of length, width and thickness. Confirmation of the accuracy of these assumed factors in a more detailed study is recommended.

Results and Discussion

For the sample of oven dried final bagasse which was collected from a diffuser factory the average values of the volume shape factors that were obtained are shown in Figure 1 and those of the surface shape factors are presented in Figure 2 for the different bagasse fractions. It can be seen from these two figures that considerable variation of the average shape factors occurs over the range of particle sizes found in the bagasse. It needs to be established whether a similar distribution of shape factors is found for all bagasse types, or whether it is affected by the processing history of the bagasse or the variety of cane used.

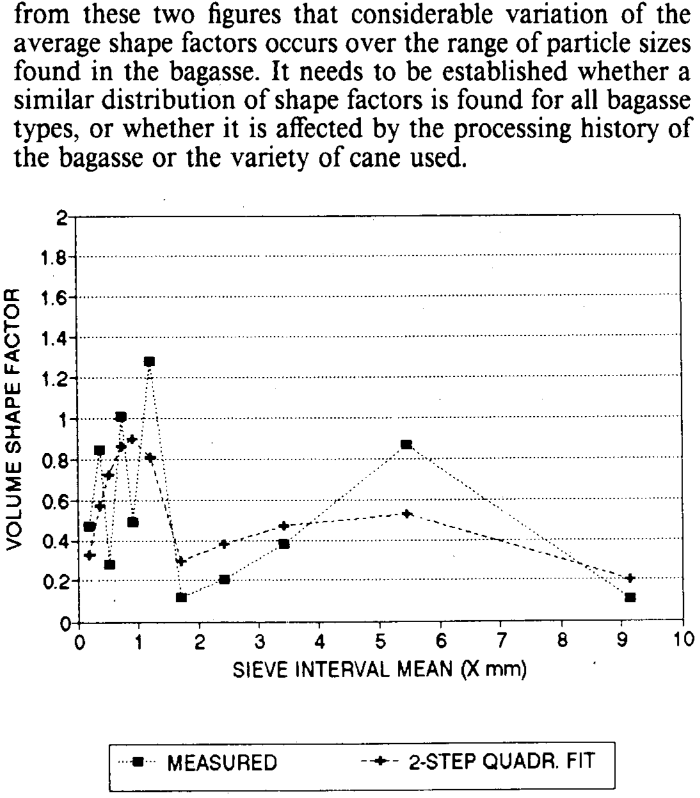

FIGURE 1 Average values of volume shape factors for bagasse fractions.

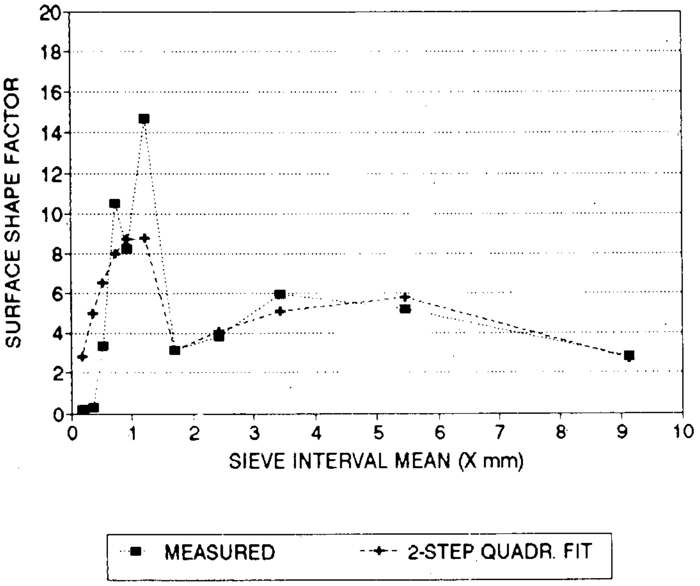

FIGURE 2 Average values of surface shape factors for bagasse fractions.

A numerical expression which relates the value of the shape factor to sieve size would be advantageous in calculations. The shape factors determined in this investigation have been approximated by quadratic expressions obtained by multilinear regression. The approximations are shown in Figures 1 and 2.

Application

The specific surface area, s, of bagasse (expressed in mm^2/g) is obtained from sieve fractionation by the following formula (Herdan, 1960, p.194), where α_s and α_v are the surface and volume shape factors respectively, ρ the particle density, m_i the mass fraction of bagasse particles retained on the ith screen and X_i the mean of the aperture sizes of the ith screen and the one above it.

$$s = \frac{\alpha_s}{\alpha_v \rho} \sum \left(\frac{m_i}{X_i}\right) \qquad (6).$$

Formula (6) assumes that the shape factors α_s and α_v are constant over the entire range of particle sizes. The data presented above have shown that this is not the case, at least for the bagasse sample analysed here. Figure 3 shows the variation of the ratio α_s/α_v measured over the full range of sieve sizes used. The measured values were approximated by a cubic expression which was evaluated by multilinear regression. The approximation is shown graphically in Figure 3 and has the form:

$$\frac{\alpha_s}{\alpha_v} = a_1 X_i + a_2 X_i^2 + a_3 X_i^3 \qquad (7).$$

For the bagasse analysed in this study the values of a_1, a_2, and a_3 were 19,79; -5,80 and 0,433 respectively. This approximation is only applicable for sieve apertures smaller than 9,5 mm.

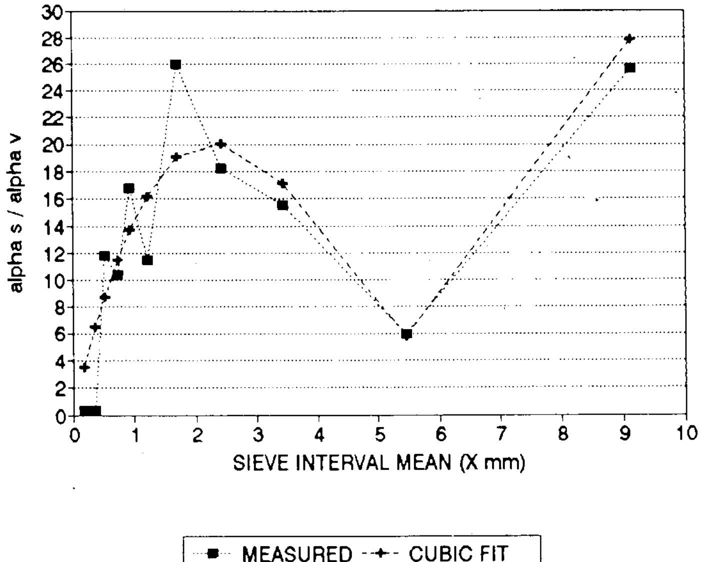

FIGURE 3 Ratios of surface: volume shape factors for bagasse fractions.

Using an average particle density value of 0,5 g/cm^3 (i.e. 0,0005 g/mm^3) for oven dried bagasse results in the following expression:

$$S = \frac{1}{0{,}0005} \sum \left(\frac{19{,}79\, X_i - 5{,}80\, X_i^2 + 0{,}433\, X_i^3}{X_i}\right) m_i \qquad (8).$$

The particle density of 0,5 g/cm^3 was obtained using a water displacement technique. Holliday (1990) measured values ranging from 0,29 to 0,57 g/cm^3 by mercury displacement for different sieve fractions of bagasse. For a given sample of oven dry bagasse (details shown in Table 1) the use of equation (8) yields a specific surface area of 24 700 mm^2/g. By comparison, the formula given in the SASTA laboratory manual for South African sugar factories (Anon., 1985), which uses a constant value of 60 for $\alpha_s/\alpha_v\rho$ and mass percentages in place of mass fractions, gives a value of 9 200 mm^2/g for the same data. The value of 60 is composed of a figure 6 representing the ratio of surface to volume shape factors for spherical particles, a factor of 100 dealing with the expression of mass fractions as percentages and a factor of 1 000 which involves conversion of particle density from g/cm^3 to g/mm^3, (60 = 6x1 000/(100x1)). It furthermore assumes a particle density of 1 g/cm^3. The difference in the value of the surface area calculated by the SASTA formula is therefore partly due to a different value of particle density used (0,5 in equation (8) and 1,0 in the SASTA formula), but mainly due to the fact that it assumes a constant value of 6 for the shape factor ratio (which is the ratio applicable to spherical particles). The work of this study has shown that there is a wide variation of shape factor values for the different size fractions obtained from bagasse and that most particles present shapes that are far from spherical.

Table 1

Calculation details of specific surface area determination

Sieve interval mm	Interval mean X_i mm	Mass fraction m_i	Calculated α_s/α_v	$(\alpha_s/\alpha_v)/X_i$	m_i/X_i
6,70 - 11,22	9,146	0,065	27,10	0,193	0,007
4,00 - 6,70	5,460	0,084	5,63	0,087	0,015
2,80 - 4,00	3,435	0,077	17,09	0,383	0,022
2,00 - 2,80	2,422	0,069	20,06	0,571	0,028
1,40 - 2,00	1,717	0,077	19,07	0,855	0,045
1,00 - 1,40	1,211	0,090	16,23	1,206	0,074
0,850 - 1,00	0,927	0,062	13,71	0,917	0,067
0,600 - 0,850	0,732	0,157	11,55	2,477	0,214
0,425 - 0,600	0,517	0,125	8,74	2,113	0,242
0,300 - 0,425	0,366	0,081	6,49	1,436	0,221
0,000 - 0,300	0,189	0,113	3,54	2,114	0,598

Conclusions

A method for the determination of shape factors of bagasse particles has been presented. The study using this method was carried out on a sample of oven dried final bagasse from a diffuser factory.

The study has shown that the average shape factor for each size fraction investigated is not constant and does not follow a linear relationship with the mean sieve aperture. A similar study on bagasse derived from a milling tandem, as well as on shredded cane should reveal if the shape of the curve approximating bagasse shape factors is essentially the same or depends largely on the type and intensity of the process to which the bagasse has been subjected and whether it is affected by cane variety. Quantitative data on bagasse shape factors, through the evaluation of specific surface area, may be useful in attempts to optimise milling, diffusion or bagasse de-watering and find application in the design of bagasse handling equipment.

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