

# THE USE OF FUZZY EXPERT SYSTEMS TO EXAMINE VAGUE AND COMPLEX PROBLEMS IN SUGAR ENGINEERING

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## Abstract

Development in engineering is often slowed by the fact that the total number of experiments required to establish a proposition usually has to be very high to guarantee the prescribed level of confidence. As a consequence, whatever knowledge is available becomes very important, and engineers often have to make their decisions using very vague experimental data. Ways of overcoming these problems, through the application of expert systems and methods of artificial intelligence, are considered. The method is illustrated through the application of fuzzy modelling to summarise vague knowledge of the effect of operating conditions on the fouling of evaporators, as measured through the overall heat transfer coefficient. Each operating condition (feed rate, steam pressures, temperature, ash and brix) is described in terms of fuzzy sets. Approximate knowledge of the heat transfer resulting from a limited set of conditions is then required along with an indication of the degree of confidence attached to each estimate. The knowledge base so created is then sufficient to produce a model of the system which can predict heat transfer values from input operating conditions. No prior knowledge of fuzzy mathematics is required.

## Introduction

There are many problems in sugar engineering that are difficult to tackle with conventional statistical and mathematical tools. The use of conventional techniques to analyse a real engineering problem requires enormous quantities of information which may be difficult or even impossible to obtain using the available resources. For this reason a model description of the object under study must be simplified when working with traditional tools. An alternate approach is to make use of the relatively new science of fuzzy simulation, which is a less intensive method of analysis but which can achieve more realistic results than conventional approaches in cases where the system that is modelled is complex and/or poorly understood (Dohnal *et al.*, 1994). The purpose of this paper is to present some of the principles of fuzzy systems with an example of an application of interest to the sugar industry to demonstrate the method.

In order for fuzzy systems to model complex processes effectively, all the available information must be used. Very uncertain or subjective knowledge, even hearsay, is valuable. The effectiveness with which uncertain knowledge is used is often the main distinction between good and bad models of the same system. Traditional mathematics has limited use for such information and little to offer to solve such tasks.

## Fuzzy mathematics

The most important feature of human problem solving is the ability to extract, from a collection of masses of data, only such items of knowledge which are relevant to the task at hand (Dohnal *et al.*, 1994). It is believed that the main reason why human thinking is so flexible and powerful is that, unlike numbers, words are not precisely defined. The

fuzzy set theory is based on the premise that the key elements in human thinking are not numbers but words. A linguistic value is a "value" that is given by words, e.g. high or low. To take as an example of a verbal variable "temperature", to quantify expert knowledge a set of verbal values is needed. Such a temperature "dictionary" could be:

*low, medium, high,* (Statement 1)

The linguistic value (e.g. high) of a specific variable (e.g. temperature) is transformed into a fuzzy set by the specification of a grade of membership between 0 and 1. To define the grade of membership two intervals have to be chosen, namely an interval of typical values and an interval of certain values. In the general case:

typical values {b, c} and

certain values {a, d}

where  $a < b < c < d$

(Statement 2)

Therefore the typical values interval is a sub-interval of the certain values interval. To make this more meaningful it is best to use an example with numbers in place of a, b, c and d. Suppose that the fuzzy set "high temperature" is fully specified by the following four numerical values:

$a = 80, b = 85, c = 100, d = 105$  (Statement 3)

Therefore the corresponding "typical" interval is {85, 100}. This means that, in the example, temperatures between 85 and 100 are considered, by the "expert", to be typical "high temperatures" and temperatures between 80 and 85 and 100 and 105, while not typical are still "high temperatures". All numerical values which are covered by the typical interval have the grade of membership equal to one. To deal with temperatures which are not typical but still "high temperatures", a piecewise linear grade of membership is used. This is shown graphically in Figure 1.

The numerical values in the two fuzzy intervals, namely {a, b} and {c, d} have grades of membership higher than zero and lower than one. Therefore they belong partially to the fuzzy set characterized by Figure 1.

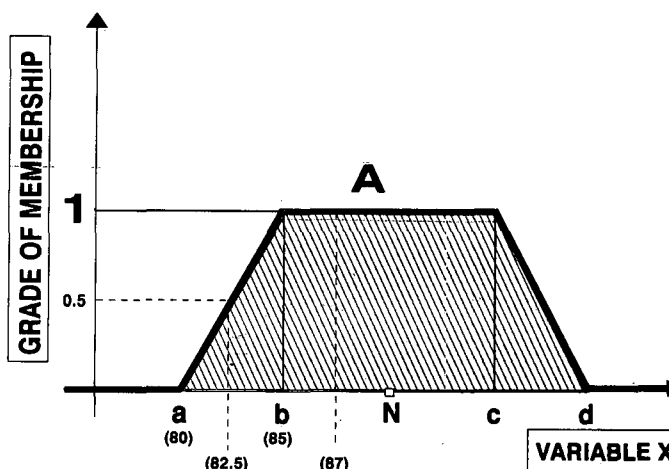


FIGURE 1 Standard shape of the grade of membership function.

In the example above (statement 3) the numerical value 82,5 would belong to the fuzzy set high temperature with a grade of membership equal to 0,5. Extensive industrial experience has confirmed that the above process is very flexible, easy to use and easy to understand. At this stage it can be seen that when  $b=c$  then the shape of the grade of membership function is a triangle and when  $a=b=c=d$  then the fuzzy set is a single number.

Suppose that in a particular case a human understanding of "medium" and "high" temperatures results in an overlap, i.e. some temperatures may be partially medium and partially high. This is shown in Figure 2. The fact that there is an overlap reflects the feeling of the engineers and their inability to quantify the values exactly. It is clear that the overlap can occur only in fuzzy intervals. It is not possible that a 100% high temperature would belong to the medium temperature with a non-zero grade of membership. For example, the temperature T shown in Figure 2 belongs to the fuzzy set "medium temperature" with the grade of membership 0,6 and it belongs to the fuzzy set "high temperature" with the grade of membership 0,1. Even if the same topic is under study, different experts can use different dictionaries because the system is ill-known. Each variable in the system of interest can be defined in terms of fuzzy sets as outlined above.

The next step is to combine the fuzzy sets into a set of conditional statements using the defined verbal dictionary, based on the experience of the engineer. Suppose that the total number of variables, n, including the intended dependent variable, is 6

$X_1, X_2, X_3, X_4, X_5, X_6$  (Statement 4)  
 and each variable has only three values (for example low, medium and high). Six variables and three values give  $3^6$  possible combinations (i.e. 729). Unlike a complete statistical factorial set of experiments, in fuzzy systems engineers will not be able to verify, reject, or assess all possible combinations. Some are not relevant, certain combinations are unknown etc. On average only approximately 2-10% of the total amount of possible combinations are known. Therefore the available data (knowledge) can be rather sparse. The fuzzy model is represented by all available combinations of fuzzy sets. The total number of combinations is m. If, for example,  $m = 3$  then a possible fuzzy model is:

Combination	$X_1$	$X_2$	$X_3$
1	low	low	medium
2	high	low	medium
3	medium	low	high
Combination	$X_4$	$X_5$	$X_6$
1	low	low	medium
2	high	medium	low
3	low	medium	low

(Statement 5)

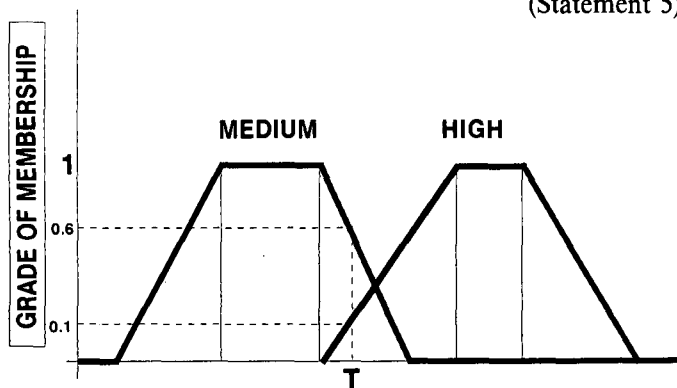


FIGURE 2 Fuzzy overlap between fuzzy sets, medium and high.

A set of all available combinations is a matrix M  
 $M (m \cdot n)$  (Statement 6)

There are m combinations and n variables making up the set of statements. One has to keep in mind that any variable can be considered as the dependent variable and any subset of variables can be chosen as a set of dependent variables. The choice of dependent and independent variables represents a certain *ad hoc* point of view only.

The fuzzy model can be used to answer the following query:

Query	$X_1$	$X_2$	$X_3$
1	Low	Low	Medium
Query	$X_4$	$X_5$	$X_6$
1	Low	Low	Unknown

(Statement 7)

The unknown value is the corresponding value of the variable  $X_6$ . Therefore the variable  $X_6$  is the dependent variable. The solution in this case is trivial. The query is identical to the first combination in statement 5 and therefore the unknown value is equal to medium. The fuzzy model gives fuzzy sets as answers.

However, the results of the fuzzy modelling may be used as constants in conventional models and any fuzzy set can be represented by a number. The numerical representation is an equivalent of the mean value in its statistical sense. One possible algorithm used to evaluate the numerical representation of a fuzzy set is a center of gravity of the corresponding area. If the fuzzy set given in Figure 1 were the answer then its numerical representation would be the value N which is the centre of gravity of the shaded area.

Now consider a case in which a query does not correspond exactly to one of the conditional statements. In such a case sophisticated reasoning (interpolating) algorithms have to be used. The details of these are outside the scope of this paper but the principle on which they are based is as follows:

Suppose that the algorithm used can determine that the query is similar to two conditional statements in the matrix which makes up the model. In other words the dependent variable required is "close to" two fuzzy sets specified by the fuzzy model matrix. These can be shown graphically in Figure 3, as fuzzy sets ABCD and EFGH respectively. Assume further that the degree of similarity in each case can be calculated. Let these be  $s_1$  and  $s_2$  (the precise definition of s is not important at present). Furthermore let  $s_2$  be greater than  $s_1$ , i.e. the second fuzzy set is considered more like the query than the first. These values can be plotted on the x axis as shown in Figure 3 and used to produce the answer, the area (fuzzy set) outlined by A W<sub>1</sub> I W<sub>3</sub> W<sub>4</sub>H. Thus the fuzzy answer is a "weighted union" of the two fuzzy sets

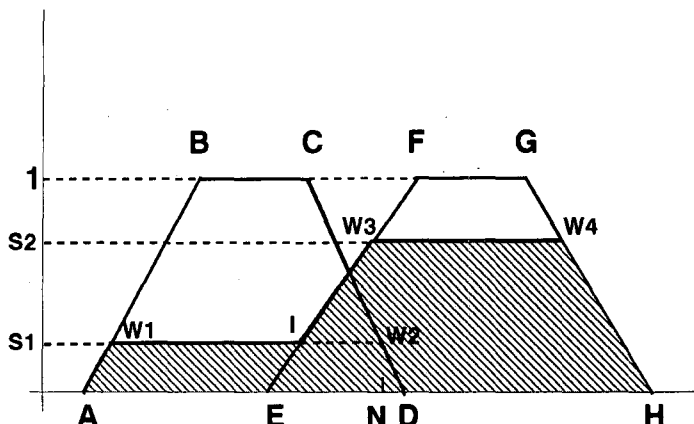


FIGURE 3 An example of a fuzzy answer.

which were considered similar to the query. The centre of gravity becomes the numerical representation of the fuzzy answer, say N shown in Figure 3.

Clearly an interpolation algorithm is essential to make use of the conditional statements. A reader who is willing to invest time in a formal study of fuzzy mathematics should consult more detailed texts for the mathematically rigorous definition of these algorithms (Bullock *et al.*, 1994; Dohnal *et al.*, 1993; Zimmermann, 1986). However, if the basic principle is clearly understood there is no need for engineers who want to solve problems to be involved in actual mathematical procedures. The team approach is inevitable and more productive.

**Example of a fuzzy model development: evaporator fouling**

There is no generally applicable methodology of fuzzy model development. However, the following sequence of steps, with an example, indicates the basic procedure involved. The example considered is related to the fouling of first effect evaporators resulting from particular operating conditions. This particular problem is one which is difficult to analyse or optimise since well defined data are difficult to obtain although a reasonable amount of anecdotal evidence is available (Reppich *et al.*, 1993). The example below is based on vague data gathered during discussions with staff at a number of mills, and information has been modified so as to be applicable to one of the first effect Kestners in the quintuple effect evaporator train at Felixton sugar mill.

*Step 1: Choose a set of relevant variables*

In choosing the necessary relevant variables the total number needs to be kept as low as possible. In this example it was felt that the fouling is a function of feed rate, steam pressure, vapour pressure, brix and ash. Furthermore, it was reasoned that the fouling would be adequately reflected in the overall heat transfer coefficient. The values estimated for the overall heat transfer coefficient should be considered relative rather than absolute. In other words fouling is represented by an unknown five-dimensional mathematical function:

$$Y = \varnothing (X_1, X_2, v. . . , X_5) \quad \text{(Statement 10)}$$

The chosen variables and units are:

Variable Description	Unit	
X <sub>1</sub> - Feed rate	[tons/hour]	
X <sub>2</sub> - Steam pressure	[kPag]	
X <sub>3</sub> - Vapour pressure	[kPag]	
X <sub>4</sub> - Sugar concentration of feed (brix)	[%]	
X <sub>5</sub> - Ash	[%]	
Y - Overall heat transfer coefficient (OHTC)	[kW/m <sup>2</sup> °C]	(Statement 11)

*Step 2: Choose a set of verbal values for each relevant variable (create a dictionary)*

Typical values are low, medium, high and acceptable. Nevertheless one should not hesitate to use jargon or personal terminology. On the other hand, one should not use too many values. In this example it was decided that the following verbal values were sufficient:

- V - Very low
  - L - Low
  - N - Normal
  - H - High
  - A - Acceptable
  - VH - Very high
- (Statement 12)

It should be noted that not every variable needs to be defined at each verbal value, since in some cases only two values were considered sufficient.

Next, by choosing the corresponding values a, b, c, d (see Figure 1), all verbal values are transferred into fuzzy sets to form a "dictionary". This is given in Table 1.

**Table 1**  
**Fuzzy dictionaries**

Variable description		Verbal values	a	b	c	d
Feed rate	X <sub>1</sub>	L (Low)	200	250	275	300
		N (Normal)	290	350	400	425
		H (High)	375	400	450	480
Steam pressure	X <sub>2</sub>	L (Low)	40	50	80	83
		H (High)	80	85	100	105
Vapour pressure	X <sub>3</sub>	L (Low)	10	20	30	50
		H (High)	50	60	80	90
Sugar concentration	X <sub>4</sub>	L (Low)	6,0	6,5	10,0	11,5
		H (High)	10,0	11,0	13,0	15,5
Non-sugar concentration	X <sub>5</sub>	N (Normal)	0,1	0,2	0,2	0,4
		H (High)	0,4	1,0	2,0	3,0
Overall heat transfer coefficient	Y	VL (Very low)	0,3	0,5	0,6	0,75
		L (Low)	0,75	1,0	1,0	1,5
		A (Acceptable)	1,0	2,0	2,5	3,0
		H (High)	2,5	3,0	3,0	3,5
		VH (Very high)	3,0	3,5	3,5	4,0

*Step 3: Generate all possible combinations of linguistic values (conditional statements)*

It is instructive, in this example, to choose a specific dependent variable. In this case, it is the OHTC. For this reason, in statement 11, "Y" is used rather than X<sub>6</sub>. An example of a "fouling" conditional statement is:

*If feed rate is LOW and steam pressure is LOW and vapour pressure is LOW and sugar concentration of feed is LOW and non-sugar concentration of feed is NORMAL then the overall heat transfer coefficient is ACCEPTABLE*

(Statement 13)

This process was carried out to produce a knowledge base of 26 conditional statements, listed in Table 2, where each line represents a conditional statement. The declaration above corresponds to the first line (No. 1) given in this table. The last column of the set of statements in Table 2 indicates how strongly it is believed that each statement is correct. If they are absolutely certain then the corresponding weight is equal to one.

*Step 4: List queries as fuzzy sets*

The verbal dictionary (Table 1) and conditional statements (Table 2) were combined using a computer program which uses an interpolating algorithm to answer queries. In this example the fuzzy model base was then used to answer the question: "What is the effect of feed rate on fouling?". Using conventional terminology, the values of the feedrate are varied while the other variables are kept constant to evaluate the following relation:

$$Y = f(X_1)$$

To find the relationship, the question is formed as a set of five queries. In each case the variables are specified as fuzzy sets in terms of a, b, c, and d as already described.

**Table 2**

Set of fuzzy combinations (conditional statements)

Statement No.	Feed rate	Pressure		Concentrations		Heat transfer coef.	Weight (confidence)
		Steam	Vapour	Sugar	Ash		
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>	Y	
1	L	L	L	L	N	A	0,5
2	N	L	L	L	N	H	0,6
3	H	L	L	L	N	VH	0,5
4	L	H	L	L	N	A	0,5
5	N	H	L	L	N	H	0,5
6	H	H	L	L	N	VH	0,5
7	N	L	H	L	N	L	0,2
8	H	H	H	L	N	A	0,5
9	H	L	L	N	N	A	0,6
10	L	H	L	N	N	A	0,4
11	N	H	L	N	N	A	0,4
12	H	H	L	N	N	L	0,2
13	H	L	H	N	N	A	0,5
14	N	L	L	L	H	L	0,5
15	H	L	L	L	H	L	0,4
16	L	L	H	L	H	L	0,3
17	N	L	H	L	H	L	0,2
18	H	L	H	L	H	L	0,1
19	N	L	L	N	H	L	0,6
20	H	L	L	N	H	L	0,8
21	L	L	L	N	H	VL	0,9
22	L	H	H	N	H	A	0,4
23	H	H	H	N	H	L	0,1
24	L	H	H	N	H	VL	0,8
25	N	H	H	N	H	L	0,3
26	H	H	H	N	H	L	0,3

For simplicity b is chosen to be equal to c. In this example all five queries have the same specification of variables X<sub>2</sub> (steam pressure), X<sub>3</sub> (vapour pressure), X<sub>4</sub> (brix) and X<sub>5</sub> (ash) and their fuzzy specifications are given in Table 3. The only difference is the definition of the first variable X<sub>1</sub> (feed rate). The fuzzy descriptions of five values of the first variable feed rate are given in Table 4.

**Table 3**

Fuzzy values of all variables that are kept constant in all five queries

Variable	a	b = c	d
X <sub>2</sub>	80	82	84
X <sub>3</sub>	32	35	37
X <sub>4</sub>	10,0	10,5	11,0
X <sub>5</sub>	1,9	2,5	3,1

**Table 4**

Fuzzy values of the feed rate X<sub>1</sub>

Query	a	b = c	d
1	250	255	260
2	295	300	305
3	300	305	310
4	320	325	330
5	395	400	405

*Step 5: Obtain the solution as a fuzzy set*

The answers to all five queries are produced as fuzzy sets and these are given in Table 5. The explanation of the first line in this table: the first query is "fuzzy similar" to the statement No. 21 (in Table 2). The similarity is s = 0,53. The similar statement No. 21 has the value VL (Very Low,) as the value of the dependent variable Y. For the second query, the interpolation algorithm has identified three similar conditional statements from Table 2. These are statement numbers 14, 19 and 21 in Table 2 and the answer is a "weighted union" of these conditional statements. The similarities are relatively low (0,12; 0,14; 0,15), and therefore the answer is not as reliable as the first one.

**Table 5**

Give fuzzy answers to queries given in Tables 3 and 4

Query	Variable Y	Statement in Table 2	S - similarity
1	VL	21	0,53
2	L	14 19 21	0,12 0,14 0,15
3	L	14 19	0,16 0,19
4	L	14 19	0,32 0,38
5	L	14 15 19 20	0,38 0,32 0,43 0,52

*Step 6: Convert the fuzzy answers to numerical values*

Since the fuzzy answers produced have to be used for future analysis, and this analysis is based on conventional mathematics, a numerical representation of fuzzy sets is required. Thus the fuzzy answers are translated back into numbers using the weighted averages, described previously in Figure 3 as follows:

Query no. Weighted average (OHTC)

1	0,53
2	0,88
3	1,11
4	1,10
5	1,10

(Statement 14)

The graph of the relation is given in Figure 4.

However, one has to keep in mind that because the original results are fuzzy, the numerical values produced have a degree of uncertainty. This can be calculated by the program to produce guaranteed minimum and maximum values given by  $\alpha$  and  $\beta$  respectively. The interval between  $\alpha$  and  $\beta$ , given below, is then the "certain" interval for each query. Plotting these lines on the graph in Figure 4 then gives an idea of the precision of the model.

Query	$\alpha$	$\beta$
1	0,3	0,75
2	0,3	1,5
3	0,75	1,5
4	0,75	1,5
5	0,75	1,5

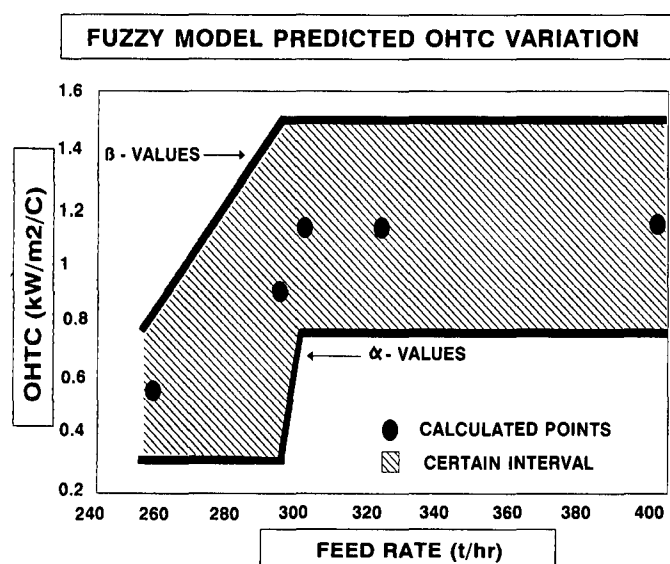


FIGURE 4 Graphic representation of the overall heat transfer coefficient as a function of the feed rate with constant values given in Table 3.

### Conclusions

The result produced by the fuzzy model gives an indication of an expected trend rather than absolute values. The model suggests that fouling will be high (low OHTC) at low flow rates but fouling will not decrease without limit as the flow rate is increased. This is in keeping with conventional thinking on the subject (Bott, 1990). Space prevents the results of other queries being published but similar queries enable the identification of areas where experimentation should be focussed. Any further results can then be used to refine the model and ultimately optimum operating conditions can be approached.

What is important is that the above relationship, and many similar relationships, simply could not be achieved without extensive and consequently expensive experimentation. Furthermore the engineer does not need to understand the intricacies of fuzzy mathematics to make use of fuzzy logic systems. The model above is still in an elementary stage and as feedback and additional information are made available

it is easily modified. It is also important to note that the above model could be modified for different applications, such as control.

However, the best formal model is always based on traditional mathematics. It is the most accurate but this accuracy must be backed by a sufficiently rich data base. Real life situations are not information intensive and therefore a broad spectrum of new calculation methods must be introduced to minimise the final information loss caused by poor management of engineering information. The best variant is a network of calculation methods. There are many different fuzzy reasoning algorithms and they have been used in the modeling of chemical engineering processes for more than fifteen years. However, to squeeze the maximum amount of information out of the available data, judgment calls based on extensive experience with fuzzy systems are still required and this is unlikely to change within the next few years.

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